FBISE

MATHEMATICS

MODEL PAPERS & GUESS PAPERS

Faderal Board Islamabad

Presented by:

Urdu Books Whatsapp Group

STUDY GROUP

9TH CLASS

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GUESS PAPER & MODEL PAPER # 01
BASED ON UNIT # 1 (Reduced Syllabus)
MATRICES AND DETERMINANTS

Unit 1	Matrices and Determinants
Exercise 1.1	Q1; Q2; Q3
Exercise 1.2	Q1; Q2; Q3; Q5; Q6
Exercise 1.3	Q1; Q2; Q5(i, v, vi, ix, x); Q6; Q7; Q8(i, ii, iii, iv)
Exercise 1.4	Q1; Q2; Q3; Q5; Q6
Exercise 1.5	Q1(i, ii); Q2(i, ii); Q3(iii, iv); Q4
Exercise 1.6	Q1(i, iii, v, vii); Q2; Q4
Review Ex 1	Q1

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itseif. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) The order of matrix $[2 \quad 1]$ is.........

(A) 2-by-1 (B) 1-by-2 (C) 1-by-1 (D) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called.........matrix.

(A) zero (B) unit (C) scalar (D) singular

(iii) Which is order of a square matrix..........

(A) 2-by-2 (B) 1-by-2 (C) 2-by-1 (D) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is.........

(A) 3-by-2 (B) 2-by-3 (C) 1-by-3 (D) 3-by-1

(vi) Product of $[x \quad y]\begin{bmatrix}2\\-1\end{bmatrix}$ is.........

(A) $[2x + y]$ (B) $[x - 2y]$ (C) $[2x - y]$ (D) $[x + 2y]$

(vii) If $\begin{vmatrix}2 & 6\\3 & x\end{vmatrix} = 0$, then x is equal to...a = ...

(A) 9 (B) -6 (C) 6 (D) -9

(viii) If $X + \begin{bmatrix}-1 & -2\\0 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then X is equal to........

(A) $\begin{bmatrix}2 & 2\\2 & 0\end{bmatrix}$ (B) $\begin{bmatrix}0 & 2\\2 & 2\end{bmatrix}$ (C) $\begin{bmatrix}2 & 0\\0 & 0\end{bmatrix}$ (D) $\begin{bmatrix}2 & 2\\0 & 2\end{bmatrix}$

(ix) $\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ is called.........matrix.

(A) Null (B) unit (C) scalar (D) Non-singular

(x) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ is called........matrix.

(A) zero (B) unit (C) scalar (D) Non-singular

(xi) Additive inverse of $\begin{bmatrix}1 & -2\\0 & -1\end{bmatrix}$ is....

(A) $\begin{bmatrix}1 & 2\\0 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}$ (C) $\begin{bmatrix}-1 & 2\\0 & 1\end{bmatrix}$ (D) $\begin{bmatrix}-1 & -2\\0 & -1\end{bmatrix}$

(xii) In matrix multiplication, in general. AB........BA.

(A) = (B) $\neq$ (C) > (D) <

(xiii) Matrix A + B may be found if order of A and B is.......

(A) greater (B) same (C) less (D) difference

(xiv) A matrix is called........matrix if number of rows and columns are equal.

(A) square (B) unit (C) scalar (D) Non-singular

(xv) If $\begin{vmatrix}2 & 6\\3 & x\end{vmatrix} = 0$, then x is equal to:

(A) 6 (B) -6 (C) -9 (D) 9

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Find the values of a, b, c and d which satisfy the matrix equation
$\begin{bmatrix}a+c & a+2b\\c-1 & 4d-6\end{bmatrix} = \begin{bmatrix}0 & -7\\3 & 2d\end{bmatrix}$; EX #1.1 Q3

(ii) The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle. ; EX #1.6 Q.2

(iii) Find the determinant of the following matrices.
(i) $A = \begin{bmatrix}-1 & 1\\2 & 0\end{bmatrix}$ (ii) $B = \begin{bmatrix}1 & 3\\2 & -2\end{bmatrix}$; EX #1.5 Q1. (i , ii)

(iv) If $A = \begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}$, $B = \begin{bmatrix}6\\5\end{bmatrix}$, find (i) AB (ii) BA (if possible) ; EX #1.4 Q2.

(v) If $2\begin{bmatrix}2 & 4\\-3 & a\end{bmatrix} + 3\begin{bmatrix}1 & b\\8 & -4\end{bmatrix} = \begin{bmatrix}7 & 10\\18 & 1\end{bmatrix}$, then find a and b. ; EX #1.3 Q7.

(vi) If $A = \begin{bmatrix}1 & -2\\3 & 4\end{bmatrix}$ and $B = \begin{bmatrix}0 & 7\\-3 & 8\end{bmatrix}$, find (i) 3A – 2B (ii) $2A^t - 3B^t$. ; EX #1.3 Q6.

(vii) The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle. ; EX #1.6 Q.4

(viii) Show whether the points with vertices (5, 2), (5, 4) and (4, −1) are vertices of an equilateral

ختم نبوت ﷺ زندہ باد **عظمت صحابہ زندہ باد**

السلام علیکم ورحمۃ اللہ وبرکاتہ:

معزز ممبران: آپ کا وٹس ایپ گروپ ایڈ من "**اردو بکس**" آپ سے مخاطب ہے۔

آپ تمام ممبران سے گزارش ہے کہ:

❖ گروپ میں صرف PDF کتب پوسٹ کی جاتی ہیں لہذا کتب کے متعلق اپنے کمنٹس / ریویوز ضرور دیں۔ گروپ میں بغیر ایڈ من کی اجازت کے کسی بھی قسم کی (اسلامی وغیر اسلامی، اخلاقی، تحریری) پوسٹ کرنا سختی سے منع ہے۔

❖ گروپ میں معزز، پڑھے لکھے، سلجھے ہوئے ممبرز موجود ہیں اخلاقیات کی پابندی کریں اور گروپ رولز کو فالو کریں بصورت دیگر معزز ممبرز کی بہتری کی خاطر ریموو کر دیا جائے گا۔

❖ کوئی بھی ممبر کسی بھی ممبر کو انباکس میں میسج، مس کال، کال نہیں کرے گا۔ رپورٹ پر فوری ریموو کر کے کاروائی عمل میں لائے جائے گی۔

❖ ہمارے کسی بھی گروپ میں سیاسی و فرقہ واریت کی بحث کی قطعاً کوئی گنجائش نہیں ہے۔

❖ اگر کسی کو بھی گروپ کے متعلق کسی قسم کی شکایت یا تجویز کی صورت میں ایڈ من سے رابطہ کیجئے۔

❖ سب سے اہم بات:

گروپ میں کسی بھی قادیانی، مرزائی، احمدی، گستاخِ رسول، گستاخِ امہات المؤمنین، گستاخِ صحابہ و خلفائے راشدین حضرت ابو بکر صدیق، حضرت عمر فاروق، حضرت عثمان غنی، حضرت علی المرتضیٰ، حضرت حسنین کریمین رضوان اللہ تعالیٰ اجمعین، گستاخ اہلبیت یا ایسے غیر مسلم جو اسلام اور پاکستان کے خلاف پراپیگنڈا میں مصروف ہیں یا ان کے روحانی وذہنی سپورٹرز کے لئے کوئی گنجائش نہیں ہے لہذا ایسے اشخاص بالکل بھی گروپ جوائن کرنے کی زحمت نہ کریں۔ معلوم ہونے پر فوراً ریموو کر دیا جائے گا۔

❖ تمام کتب انٹرنیٹ سے تلاش / ڈاؤنلوڈ کر کے **فری آف کاسٹ** وٹس ایپ گروپ میں شیئر کی جاتی ہیں۔ جو کتاب نہیں ملتی اس کے لئے معذرت کر لی جاتی ہے۔ جس میں محنت بھی صَرف ہوتی ہے لیکن ہمیں آپ سے صرف دعاؤں کی درخواست ہے۔

❖ عمران سیریز کے شوقین کیلئے علیحدہ سے عمران سیریز گروپ موجود ہے۔

❖ **لیڈیز کے لئے الگ گروپ کی سہولت موجود ہے جس کے لئے ویریفکیشن ضروری ہے۔**

❖ اردو کتب / عمران سیریز یا سٹڈی گروپ میں ایڈ ہونے کے لئے ایڈ من سے وٹس ایپ پر بذریعہ میسج رابطہ کریں اور جواب کا انتظار فرمائیں۔ برائے مہربانی اخلاقیات کا خیال رکھتے ہوئے موبائل پر کال یا ایم ایس کرنے کی کوشش ہر گز نہ کریں۔ ورنہ گروپس سے تو ریموو کیا ہی جائے گا بلاک بھی کیا جائے گا۔

نوٹ: ہمارے کسی گروپ کی کوئی فیس نہیں ہے۔ سب فی سبیل اللہ ہے

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پاکستان زندہ باد **پاکستان پائندہ باد**

اللہ تبارک تعالیٰ ہم سب کا حامی وناصر ہو

(x) In the ΔABC, $m\angle B = 70^\circ$ and $m\angle C = 45^\circ$. Which of the sides of the triangle is longest and which is the shortest? ; EX #13.1 ; Q.3

(xi) The three sides of a triangle are of measure 8, x and 17 respectively. For what value of x will it become base of a right angled triangle? ; EX #15; Q.3

(xii) Construct the following $\Delta's$ ABC. Draw the bisectors of their angles and verify their concurrency. ; $m\overline{AB} = 4.5$ cm, $m\overline{BC} = 3.1$ cm, $m\overline{AC} = 5.2$ cm ; EX #17.2 Q.1;(i)

(xiii) The right bisectors of the three sides of a triangle are concurrent. ; Theorem # 12.1.3

(xiv) The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm, 1.4 cm and 1.6 cm. Find the lengths of its medians. ; EX #11.4 ; Q.1

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q3. Prove that mid-point of the hypotenuse of a right triangle is equidistant from its three vertices $P(-2,5)$, $Q(1,3)$ and $R(-1,0)$. ; EX #9.3 ; Q.3

Q.4 If in any correspondence of two triangles, two angles and one side of a triangle are congruent to the corresponding two angles and one side of the other, the triangles are congruent. (A.S.A $\cong$ A.S.A) ; Theorem # 10.1.1

Q.5 Construct a quadrilateral ABCD, having
$m\overline{AB} = m\overline{AC} = 5.3\ cm$, $m\overline{BC} = m\overline{CD} = 3.8\ cm$ and $m\overline{AD} = 2.8\ cm$. ; EX #17.3 Q.1

Q.6 The line segment that joins the mid-points of two sides of a triangle is parallel to the third side and is equal to one-half of its length. ; Theorem # 11.1.3

Q.7 If three or more parallel lines make segments congruent on one transversal, they also make congruent segments on any other transversal. ; Theorem # 11.1.5

SOLUTION OF GUESS PAPER & MODEL PAPER # 1 (Reduced Syllabus)

SECTION- A (MCQs)

i. B	ii. C	iii. A	iv. B	v. A	vi. C
vii. A	viii. D	ix. A	x. B	xi. C	xii. B
xiii. B	xiv. A	xv. D			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Find the values of a, b, c and d which satisfy the matrix equation
$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$; EX #1.1 Q3

Solution: As, $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

By comparing the corresponding elements

So, $a + c = 0 \Rightarrow a = -c$ ---------- (i)

$a + 2b = -7 \Rightarrow 2b = -(a + 7)$ ---------- (ii)

$c - 1 = 3 \Rightarrow c = 3 + 1 \Rightarrow c = 4$ ---------- (iii)

By putting the value of "c" in equation (i), we will get

$a = -4$ ---------- (iv)

By putting the value of "a" in equation (ii), we will get

$2b = -(-4 + 7)$ ...

(ii) The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle. ; EX #1.6 Q.2

Solution: By Cramer's rule:

Let length of rectangle is x cm and width of rectangle is y cm.

According to given condition $x = 4y$

or $x - 4y = 0$ ------- (i)

Perimeter = 150 cm

Perimeter = $2(x + y)$ = 150

or $x + y = 75$ ------ (ii)

By solving (i) and (ii), we get $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 75 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -4 \\ 1 & 1 \end{vmatrix} = 1 \times 1 - (-4) \times 1 = 1 + 4 = 5 \neq 0$

$A_x = \begin{bmatrix} 0 & -4 \\ 75 & 1 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 0 & -4 \\ 75 & 1 \end{vmatrix} = 0 \times 1 - (-4) \times (75) = 0 + 300 = 300$

$A_y = \begin{bmatrix} 1 & 0 \\ 1 & 75 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 1 & 0 \\ 1 & 75 \end{vmatrix} = 1 \times 75 - 0 \times 1 = 75 - 0 = 75$

$x = \dfrac{|A_x|}{|A|} = \dfrac{300}{5} = 60$

$y = \dfrac{|A_y|}{|A|} = \dfrac{75}{5} = 15$

$x = 60,\ y = 15$ So length = x = 60 cm ; width = y = 15 cm.

(iii) Find the determinant of the following matrices.

(i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$; EX #1.5 Q1. (i, ii)

Solution: (i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

Determinant of matrix A is calculated as:

$|A| = \det A = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = (-1) \times 0 - 2 \times 1 = 0 - 2 = -2$

(ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

Determinant of matrix B is calculated as:

$|B| = \det B = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} = 1 \times (-2) - 2 \times 3 = -2 - 6 = -8$

(iv) If $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$, find (i) AB (ii) BA (if possible) ; EX #1.4 Q2.

Solution: (i) $AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \times 6 + 0 \times 5 \\ (-1) \times 6 + 2 \times 5 \end{bmatrix} = \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix} = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

So. $AB = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$

(ii) BA

BA is not possible (because number of columns of B is not equal to number of rows of A)

(v) If $2\begin{bmatrix} 2 & 4 \\ -3 & a \end{bmatrix} + 3\begin{bmatrix} 1 & b \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$, then find a and b. ; EX #1.3 Q7.

$= \begin{bmatrix} 2\times 2 & 2\times(4) \\ 2\times(-3) & 2\times a \end{bmatrix} + \begin{bmatrix} 3\times 1 & 3\times b \\ 3\times 8 & 3\times(-4) \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ -6 & 2a \end{bmatrix} + \begin{bmatrix} 3 & 3b \\ 24 & -12 \end{bmatrix}$

$= \begin{bmatrix} 4+3 & -8+3b \\ (-6)+24 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix}$

By equating it with R.H.S, we have: $\begin{bmatrix} 7 & 8+3b \\ 18 & 2a-12 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 18 & 1 \end{bmatrix}$

By comparing corresponding elements

$8 + 3b = 10 \Rightarrow 3b = 10 - 8 \Rightarrow 3b = 2 \Rightarrow b = \frac{2}{3}$ ------- (eq-1)

$2a - 12 = 1 \Rightarrow 2a = 1 + 12 \Rightarrow 2a = 13 \Rightarrow a = \frac{13}{2}$ ---------- (eq-2)

From equations "1" and "2", we get; $a = \frac{13}{2}$ and $b = \frac{2}{3}$

(vi) If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$, find (i) $3A - 2B$ (ii) $2A^t - 3B^t$. ; EX #1.3 Q6.

Solution: (i) $3A - 2B$

$= 3 \times \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2 \times \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$

$= \begin{bmatrix} 3\times 1 & 3\times(-2) \\ 3\times 3 & 3\times 4 \end{bmatrix} - \begin{bmatrix} 2\times 0 & 2\times 7 \\ 2\times(-3) & 2\times 8 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$

$= \begin{bmatrix} 3-0 & (-6)-(14) \\ 9-(-6) & 12-16 \end{bmatrix} = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$

So, $3A - 2B = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$

(ii) $2A^t - 3B^t$

Solution: $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$2A^t = 2 \times \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, $3B^t = 3 \times \begin{bmatrix} 0 & -3 \\ 7 & 8 \end{bmatrix}$

$2A^t = \begin{bmatrix} 2\times 1 & 2\times 3 \\ 2\times(-2) & 2\times 4 \end{bmatrix}$, $3B^t = \begin{bmatrix} 3\times 0 & 3\times(-3) \\ 3\times 7 & 3\times 8 \end{bmatrix}$

$2A^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}$, $3B^t = \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix}$

$2A^t - 3B^t = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -9 \\ 21 & 24 \end{bmatrix} = \begin{bmatrix} 2-0 & 6-(-9) \\ -4-21 & 8-24 \end{bmatrix} = \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$

So, $2A^t - 3B^t = \begin{bmatrix} 2 & 15 \\ -25 & -16 \end{bmatrix}$

(vii) The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle. ; EX #1.6 Q.4

Solution: By Cramer's rule:

Let the each equal angle be x° cm and the third angle be y° cm. According to given condition:

$\therefore$ $2x - 16 = y$

or $2x - y = 16$ ------- (i)

and $2x + y = 180^\circ$

or $2x + y = 180$ ------- (ii)

or $\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$

$2x - y = 16$; $2x + y = 180$

$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 16 & -1 \\ 180 & 1 \end{vmatrix} = 16 \times 1 - (-1) \times (180) = 16 + 180 = 196$

$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 80 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 2 & 16 \\ 2 & 80 \end{vmatrix} = 2 \times 180 - 16 \times 2 = 360 - 32 = 328$

$x = \dfrac{|A_x|}{|A|} = \dfrac{196}{4} = 49$

$y = \dfrac{|A_y|}{|A|} = \dfrac{328}{4} = 82$

$\Rightarrow \quad x = 49, \quad y = 82$

$x + y + z = 180^0 \Rightarrow 49^0 + 82^0 + z = 180^0 \Rightarrow z = 180^0 - 49^0 - 82^0 = 49^0$

So the angles are 49°, 49°, 82°

(viii) Show whether the points with vertices $(5,2)$, $(5,4)$ and $(4,-1)$ are vertices of an equilateral triangle or an isosceles triangle? ; EX #9.2 ; Q.1

Solution: Let the points be A(5,2), B(5,4) and C(-4,1).

$|AB| = \sqrt{(5-5)^2 + (4+2)^2} = \sqrt{(0)^2 + (6)^2} = \sqrt{0 + 36} = 6$

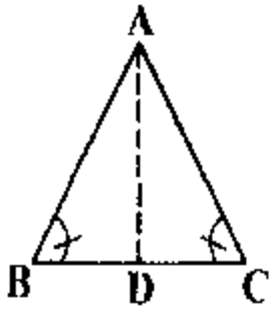
$|BC| = \sqrt{(5+4)^2 + (4-1)^2} = \sqrt{(9)^2 + (3)^2} = \sqrt{81 + 9} = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

$|CA| = \sqrt{(5+4)^2 + (-2-1)^2} \quad \sqrt{(9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}$

$As \quad |BC| = |CA| = 3\sqrt{10}$

Since two sides are equal therefore the triangle is formed is an isosceles triangle.

(ix) If two angles of a triangle are congruent, then the sides opposite to them are also congruent. ; Theorem # 10.1.2



Solution:

Given: In ΔABC,

$\angle B \cong \angle C$

To Prove:

$\overline{AB} \cong \overline{AC}$

Construction:

Draw the bisector of $\angle A$, to meet $\overline{BC}$ at point D.

Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore \Delta ABD \cong \Delta ACD$	A.A.S. ≅ A.A.S
Hence $\overline{AB} \cong \overline{AC}$	Corresponding angles of congruent triangles

(x) In the ΔABC, m∠B = 70° and m∠C = 45. Which of the sides of the triangle is longest and which is the shortest? ; EX #13.1 ; Q.3

Solution: $m\angle B = 70^\circ$; $m\angle C = 45^\circ$

$m\angle A + m\angle B + m\angle C = 180^\circ \Rightarrow m\angle A + 70^\circ + 45^\circ = 180^\circ$

$m\angle A + 115^\circ = 180^\circ \Rightarrow m\angle A = 180^\circ - 115^\circ = 65^\circ$

Since the largest angle is B. So the longest side is opposite to B is $\overline{AC}$ (Longest)

Since the smallest angle is C. So the shortest side is opposite to C is $\overline{AB}$ (Shortest)

By Pythagoras Theorem; $(hypotenus)^2 = (base)^2 + (perpendicular)^2$

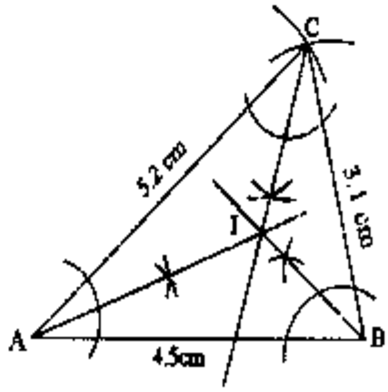
$(17)^2 = (x)^2 + (8)^2 \Rightarrow 289 = x^2 + 64$

$x^2 = 289 - 64 = 225 \Rightarrow x = \sqrt{225} = 15$

(xii) Construct the following $\Delta's$ ABC. Draw the bisectors of their angles and verify their concurrency. ; $m\overline{AB} = 4.5$ cm, $m\overline{BC} = 3.1$ cm, $m\overline{AC} = 5.2$ cm ; EX #17.2 Q.1;(i)

Solution:

Construction:



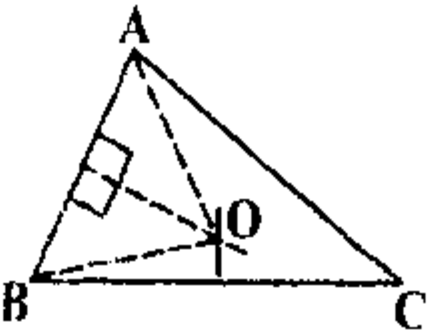
(i) Take $m\overline{AB} = 4.5$ cm.
(ii) With B as centre and radius $m\overline{BC} = 3.1$ cm draw an arc.
(iii) With centre A and radius $m\overline{AC} = 5.2$ cm draw another arc which intersects the first arc at C.
(iv) Join $\overline{CA}$ and $\overline{CB}$ to complete the ΔABC.
(v) Draw bisectors of $\angle B$ and $\angle C$ meeting each other at the point I.
(vi) Now draw the bisector of the third $\angle A$.
(vii) We observe that the third angle bisector also passes through the point I.
(viii) Hence the angle bisectors of the ΔABC are concurrent at I.

(xiii) The right bisectors of the three sides of a triangle are concurrent. ; Theorem # 12.1.3

Solution: Given: ABC is a triangle

To Prove:

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.



Construction:

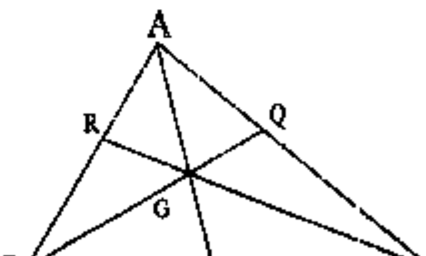
Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$, which meet each other at the point O. Join O to A, B and C.

Proof:

Statements	Reasons
$\overline{OA} \cong \overline{OB}$ .... (i)	Each point on right bisector of a segment is equidistant from its end point.
$\overline{OB} \cong \overline{OC}$ .... (ii)	From (i)
$\overline{OA} \cong \overline{OC}$ .... (iii)	From (i) and (ii)
(iv) Point O is on the right bisector of $\overline{CA}$.	O is equidistant from A and C.
(v) Point O is on the right bisector of $\overline{AB}$ and $\overline{BC}$.	Construction
Thus, the right bisectors of the three sides of a triangle are concurrent.	From (iv) and (v)

(xiv) The distance of the point of concurrency of the medians of a triangle from its vertices are respectively 1.2 cm, 1.4 cm and 1.6 cm. Find the lengths of its medians. ; EX #11.4 ; Q.1

Solution:



Let ABC be triangle with the point of concurrency of medians at G.

$m\overline{AG} = 1.2$ cm, $m\overline{BG} = 1.4$ cm and $m\overline{CG} = 1.6$ cm

$m(\overline{AP}) = \frac{3}{2}(m\overline{AG}) = \frac{3}{2} \times 1.2 = 1.8$ cm

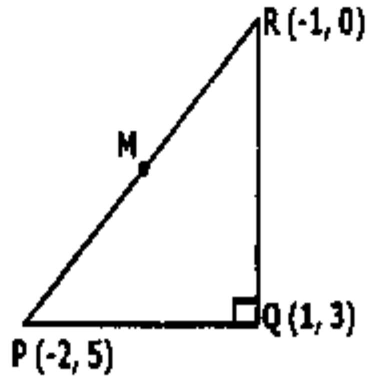
$m\overline{BQ} = \frac{3}{2}(m\overline{BG}) = \frac{3}{2} \times 1.4 = 2.1$ cm

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q3. Prove that mid-point of the hypotenuse of a right triangle is equidistant from its three vertices $P(-2,5)$, $Q(1,3)$ and $R(-1,0)$. ; EX #9.3 ; Q.3

Solution: $P(-2,5), Q(1,3), R(-1,0)$



$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{(-2-1)^2 + (5-3)^2} = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$

$|QR| = \sqrt{(1+1)^2 + (3-0)^2} == \sqrt{(-2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$

$|PR| = \sqrt{(-2+1)^2 + (5-0)^2}$

$= \sqrt{(-1)^2 + (5)^2} = \sqrt{1+25} = \sqrt{26}$

$|PR|^2 = 26 = |PQ|^2 + |QR|^2$

$\therefore \overline{PR}$ is hypotenuse

Mid point of hypotenuse PR is $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M\left(\frac{-2-1}{2}, \frac{5+0}{2}\right) = \left(-\frac{3}{2}, \frac{5}{2}\right)$

$\therefore \quad |MP|^2 = |MR|^2$

$M\left(-\frac{3}{2}, \frac{5}{2}\right)$; $R(-1,0)$

$$|MR| = \sqrt{\left(-\frac{3}{2}+1\right)^2 + \left(\frac{5}{2}-0\right)^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{25}{4}} = \frac{\sqrt{26}}{2}$$

$M\left(-\frac{3}{2}, \frac{5}{2}\right)$; $Q(1,3)$

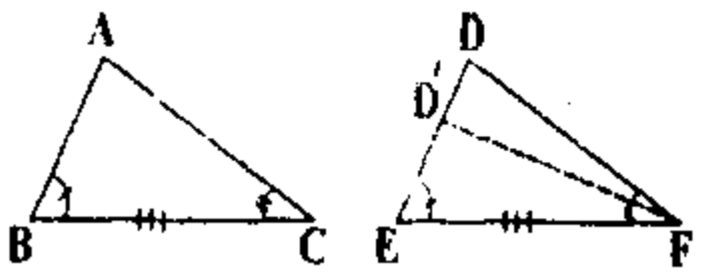
Now $|MQ| = \sqrt{\left(-\frac{3}{2}-1\right)^2 + \left(\frac{5}{2}-3\right)^2} = \sqrt{\left(-\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

$$|MQ| = \sqrt{\frac{25}{4} + \frac{1}{4}} = \sqrt{\frac{26}{4}} = \frac{\sqrt{26}}{2} = |MP| = |MR|$$

Hence M the mid point of hypotenuse is equidistant from the three vertices of the triangle PQR.

Q.4 If in any correspondence of two triangles, two angles and one side of a triangle are congruent to the corresponding two angles and one side of the other, the triangles are congruent. (A.S.A $\cong$ A.S.A) ; Theorem # 10.1.1

Solution:



Given: In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

Proof:

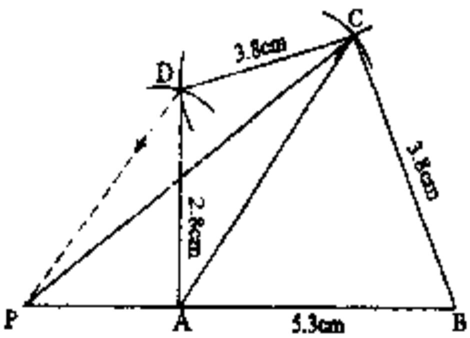
Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta D'EF$	
$\overline{AB} \cong \overline{D'E}$ .... (i)	Construction / Supposition
$\overline{BC} \cong \overline{EF}$ .... (ii)	Given
$\angle B \cong \angle E$ .... (iii)	Given
$\therefore \Delta ABC \cong \Delta D'EF$	S.A.S. Postulate
So, $\angle C \cong \angle D'EF$	Corresponding angles of congruent triangles
But $\angle C \cong \angle DFE$	Given
$\therefore \angle DFE \cong \angle D'FE$	Both congruent to $\angle C$
This is possible only if D and D' are the same points.	
So, $\overline{AB} \cong \overline{DE}$ .... (iv)	Proved that D and D' are the same points.
Thus from (ii), (iii) and (iv), we have	
$\Delta ABC \cong \Delta DEF$	S.A.S. postulate

Q.5 Construct a quadrilateral ABCD, having
$m\overline{AB} = m\overline{AC} = 5.3\,cm, m\overline{BC} = m\overline{CD} = 3.8\,cm$ and $m\overline{AD} = 2.8\,cm$. ; EX #17.3 Q.1

Solution:

Construction:

(i) With centre at A and B radius 5.3 cm draw an arc.
(ii) Take $m\overline{AB}$=5.3 cm.
(iii) With centre at B and radius 3.8 cm draw another arc to cut the first arc at D.
(iv) Join $\overline{BC}$ and $\overline{AC}$.
(v) With centre at C and radius 3.8 cm draw an arc.
(vi) With centre at A and radius 2.8 c draw another arc to cut the first arc at D.
(vii) Join $\overline{AD}$ and $\overline{DC}$ to complete the quadrilateral ABCD.
(viii) Through D draw $\parallel CA$ meeting BA produced at P.
(ix) Join PC.
(x) The ΔPBC is the required triangle.

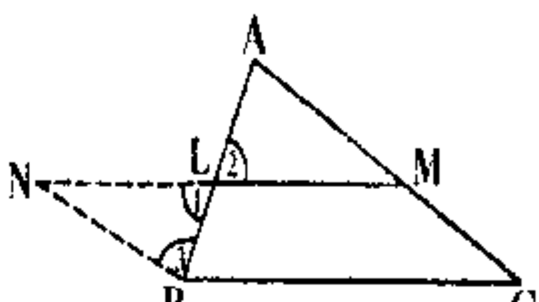


Q.6 The line segment that joins the mid-points of two sides of a triangle is parallel to the third side and is equal to one-half of its length. ; Theorem # 11.1.3

Solution:

Given:

In ΔABC, the mid-points of $\overline{AB}$ and $\overline{AC}$ are L and M respectively.



To Prove:

$\overline{LM} \parallel$

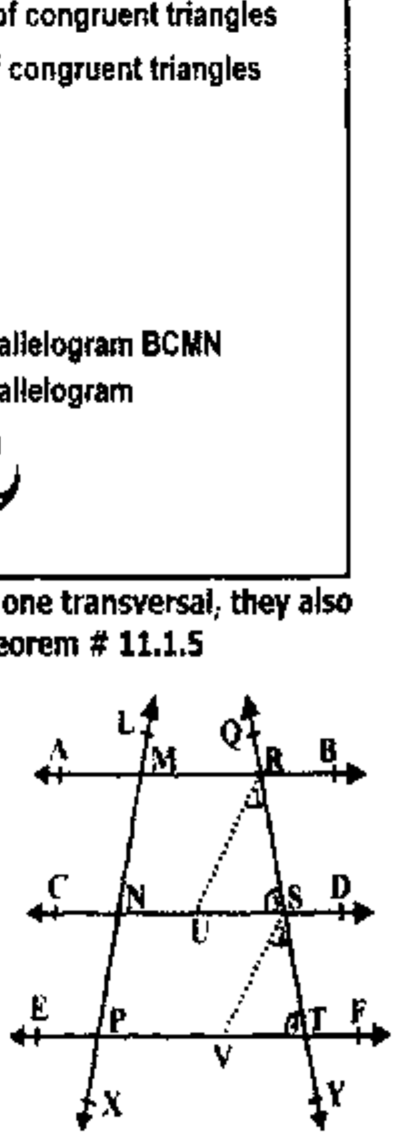
Construction:

Proof:

Statements	Reasons
In $\Delta BLN \leftrightarrow \Delta ALM$	
$\overline{BL} \cong \overline{AL}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{NL} \cong \overline{ML}$	Construction
$\therefore \Delta BLN \cong \Delta ALM$	S.A.S postulate
and $\angle A \cong \angle 3$ ...... (i)	Corresponding angles of congruent triangles
$\overline{NB} \cong \overline{AM}$ ...... (ii)	Corresponding sides of congruent triangles
$\overline{NB} \parallel \overline{AM}$	From (i)
$\Rightarrow \overline{NB} \parallel \overline{MC}$ ...... (iii)	M is mid-point of $\overline{AC}$
$\overline{MC} \cong \overline{AM}$ ...... (iv)	Given
$\overline{NB} \cong \overline{MC}$ .... (v)	From (ii) and (iv)
$\therefore$ BCMN is a parallelogram	From (iii) and (v)
$\overline{BC} \parallel \overline{LM}$ or $\overline{BC} \parallel \overline{NL}$	Opposite sides of a parallelogram BCMN
$\overline{BC} \cong \overline{MN}$ .... (vi)	Opposite sides of a parallelogram
$m\,\overline{LM} = \frac{1}{2}\, m\,\overline{NM}$ .... (vii)	Construction
Thus $m\,\overline{LM} = \frac{1}{2}\, m\,\overline{BC}$	From (vi) and (vii)

Q.7 If three or more parallel lines make segments congruent on one transversal, they also make congruent segments on any other transversal. ; Theorem # 11.1.5

Solution:



Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$

$\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P respectively, such that $\overline{MN} \cong \overline{NP}$. $\overleftrightarrow{QY}$ intersects them at points R,S and T respectively.

To Prove:

$\overline{RS} \cong \overline{ST}$

Construction:

From R, draw $\overline{RU} \parallel \overline{LX}$, which meets $\overline{CD}$ at U. From S, draw $\overline{SV} \parallel \overline{LX}$ which meets $\overline{EF}$ at V and according to the figure the names of the angles are $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	Reasons
MNUR is a parallelogram	$\overline{RU} \parallel \overline{LX}$ (construction) $\overline{AB} \parallel \overline{CD}$ (given)
$\overline{MN} \cong \overline{RU}$ .... (i)	Opposite sides of parallelogram
Similarly	
$\overline{NP} \cong \overline{SV}$ .... (ii)	
But $\overline{MN} \cong \overline{NP}$ .... (iii)	Given
$\therefore \overline{RU} \cong \overline{SV}$	From (i), (iii) and (iii)
Also $\overline{RU} \parallel \overline{SV}$	Each one $\parallel \overleftrightarrow{LX}$ (construction)
	Corresponding angles

$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Proved
$\angle 3 \cong \angle 4$	Proved
$\therefore \quad \Delta RUS \cong \Delta SVT$	S.A.A ≅ S.A.A
And $\quad \overline{RS} \cong \overline{ST}$	Corresponding sides of congruent triangles

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Find the order of the following matrices.

$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, $F = \begin{bmatrix} 2 \end{bmatrix}$,

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, $H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$; **EX #1.1 Q1**

Solution:

Order of Matrix:

The number of rows and columns in a matrix specifies its order.

(i) Matrix A has two rows and two columns so it's order = number of rows × number of columns = 2-by-2.
(ii) Matrix B has two rows and two columns so it's order = number of rows × number of columns = 2-by-2.
(iii) Matrix C has one row and two columns so it's order = number of rows × number of columns = 1-by-2.
(iv) Matrix D has three rows and one column so it's order = number of rows × number of columns = 3- by-1.
(v) Matrix E has three rows and two columns so it's order = number of rows × number of columns = 3-by-2.
(vi) Matrix F has only one row and one column so it's order = number of rows × number of columns = 1-by-1.
(vii) Matrix G has three rows and three columns so it's order = number of rows × number of columns = 3-by-3.
(viii) Matrix B has two rows and three columns so it's order = number of rows × number of columns = 2-by-3.

Q2. Which of the following matrices are equal?

$A = \begin{bmatrix} 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, $E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$,

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, $H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 3 & 3+2 \end{bmatrix}$, $J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$; **EX #1.1 Q2**

Solution:

Matrices are said to be equal if

(i) They are of same order, (ii) Their corresponding entries are equal. So, according to this definition

Ans. (a) **Matrices A and C are equal $A = C$.**
(b) **Matrices B and I are equal $B = I$.**
(c) **Matrices E, H and J are equal $E = H = J$.**
(d) **Matrices F and G are equal $F = G$.**

Q1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 0 \\ \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

Solution: Matrix A is a null matrix (because it's all entities are zero).
Matrix B is a row matrix (because it has only one row).
Matrix C is a column matrix (because it has only one column).
Matrix D is a diagonal matrix (because it's diagonal entities are 1).
Matrix E is a null matrix (because it's all entities are 0).
Matrix F is a column matrix (because it has only one column)

Q2. From the following matrices, identify

(a) Square matrices **(b)** Rectangular matrices **(c)** Row matrices
(d) Column matrices **(e)** Identity matrices **(f)** Null matrices

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$ (vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ix) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$; **EX #1.2 Q.2**

Solution:

(a) (iii),(iv) and (viii) are square matrices because the number of rows are equal to number of columns.
(b) (i),(ii),(v),(vi),(vii),(ix) are rectangular matrices because their rows and columns are not equal.
(c) (vi) is a row matrix because it has only one row.
(d) (ii) and (vii) are column matrices because they have only one column.
(e) (iv) is a identity matrix as well because its diagonal elements are "1".
(f) (ix) is a null matrix because its each entity is zero.

Q3. From the following matrices, identify diagonal, scalar and unit (identity) matrices.

$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$,

$E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix}$; **EX #1.2 Q3**

Solution: Matrix A is a scalar matrix (because its diagonal entities are same).
Solution: Matrix B is a diagonal matrix (because its diagonal entities are non-zero and non diagonal entities are zero).
Solution: Matrix C is a identity matrix (because its diagonal entities are 1).
Solution: Matrix D is a diagonal matrix (because its one diagonal entity is non-zero and non-diagonal entities are zero).
Solution: Matrix E is a scalar matrix (because its diagonal entities are same).

Q5. Find the transpose of each of the following matrices:

$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$,

$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$, $E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; **EX #1.2 Q5.**

Solution: Transpose of a matrix is obtained by converting all the columns of that matrix to the rows and all the rows to the columns So, according to the definition;

(i) $A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$ (ii) $B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$ (iii) $C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$

(iv) $D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$ (v) $E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$ (vi) $F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Q6. Verify that if B = [1 1 ... then (ii) $(B^t)^t = B$; EX #1.2 Q6 (ii)

Taking transpose of B^t, we will get $(B^t)^t = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix} = B$

Hence proved: $(B^t)^t = B$

Q1. Which of the following matrices are conformable for addition?

$A = \begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}$, $B = \begin{bmatrix}3\\1\end{bmatrix}$, $C = \begin{bmatrix}1 & 0\\2 & -1\\1 & -2\end{bmatrix}$,

$D = \begin{bmatrix}2+1\\3\end{bmatrix}$ $E = \begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$, $F = \begin{bmatrix}3 & 2\\1+1 & -4\\3+2 & 2+1\end{bmatrix}$; EX #1.3 Q1

Solution: Matrices of same order are conformable for addition. So, according to this definition;

(i) Matrices A and E are conformable for addition (because both have order 2-by-2).
(ii) Matrices B and D are conformable for addition (because both have order 1-by-1).
(iii) Matrices C and F are conformable for addition (because both have order 3-by-2).

Q2. Find the additive inverse of following matrices.

$A = \begin{bmatrix}2 & 4\\-2 & 1\end{bmatrix}$, $B = \begin{bmatrix}1 & 0 & -1\\2 & -1 & 3\\3 & -2 & 1\end{bmatrix}$, $C = \begin{bmatrix}4\\-2\end{bmatrix}$

$D = \begin{bmatrix}1 & 0\\-3 & -2\\2 & 1\end{bmatrix}$, $E = \begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$, $F = \begin{bmatrix}\sqrt{3} & 1\\-1 & \sqrt{2}\end{bmatrix}$; EX #1.3 Q2.

Solution:

The additive inverse of a matrix is obtained by changing the sign of each entity So, according to the definition;

(i) Additive inverse of A = - A = $\begin{bmatrix}-2 & -4\\2 & -1\end{bmatrix}$ (ii) Additive inverse of B = - B = $\begin{bmatrix}-1 & 0 & 1\\-2 & 1 & -3\\-3 & 2 & -1\end{bmatrix}$

(iii) Additive inverse of C = - C = $\begin{bmatrix}-4\\2\end{bmatrix}$ (iv) Additive inverse of D = - D = $\begin{bmatrix}-1 & 0\\3 & 2\\-2 & -1\end{bmatrix}$

(v) Additive inverse of E = - E = $\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$ (vi) Additive inverse of F = - F = $\begin{bmatrix}-\sqrt{3} & -1\\1 & -\sqrt{2}\end{bmatrix}$

Q5. For the matrices $A = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\0 & 2 & -1\end{bmatrix}$, $B = \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and** $C = \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$ **verify the following rules.**

(i) $A + C = C + A$ (v) $(C - B) + A + C + (A - B)$ (vi) $2A + B = A + (A + B)$
(ix) $A + (B - C) = (A - C) + B$ (x) $2A + 2B = 2(A + B)$; EX #1.3 Q5. (i , v, vi , ix , x)

Solution: (i) $A + C = C + A$

$$\text{L.H.S} = A + C = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1+(-1) & 2+0 & 3+0\\2+0 & 3+(-2) & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix} = \begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix} \quad \text{------------ (1)}$$

$$\text{R.H.S} = C + A = \begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & (-2)+3 & 3+1\\1+1 & 1+-1 & 2+0\end{bmatrix} = \begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix} \quad \text{------------ (2)}$$

From "1" and "2", it is proved that: $A + C = C + A$

L.H.S = $(C-B)+A = \left(\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right) + \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$

$= \begin{bmatrix}-1-1 & 0-(-1) & 0-1\\ 0-2 & -2-(-2) & 3-2\\ 1-3 & 1-1 & 2-3\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} = \begin{bmatrix}-2 & 1 & -1\\ -2 & 0 & 1\\ -2 & 0 & -1\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}$

$= \begin{bmatrix}-2+1 & 1+2 & -1+3\\ -2+2 & 0+3 & 1+1\\ -2+1 & 0+(-1) & -1+0\end{bmatrix} = \begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$ ——— (1)

R.H.S = $C+(A-B) = \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$

$= \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \begin{bmatrix}1-1 & 2-(-1) & 3-1\\ 2-2 & 3-(-2) & 1-2\\ 1-3 & -1-1 & 0-3\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix} + \begin{bmatrix}0 & 3 & 2\\ 0 & 5 & -1\\ -2 & -2 & -3\end{bmatrix}$

$= \begin{bmatrix}-1+0 & 0+3 & 0+2\\ 0+0 & -2+5 & 3+(-1)\\ 1+(-2) & 1+(-2) & 2+(-3)\end{bmatrix} = \begin{bmatrix}-1 & 3 & 2\\ 0 & 3 & 2\\ -1 & -1 & -1\end{bmatrix}$ ——— (2)

From "1" and "2", It is proved that: $(C-B)+A = C+(A-B)$

(vi) $2A+B = A+(A+B)$

Solution: $2A+B = A+(A+B)$

L.H.S = $2A+B = 2\times\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$

$= \begin{bmatrix}2\times1 & 2\times2 & 2\times3\\ 2\times2 & 2\times3 & 2\times1\\ 2\times1 & 2\times(-1) & 2\times0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix} = \begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$

$= \begin{bmatrix}2+1 & 4+(-1) & 6+1\\ 4+2 & 6+(-2) & 2+2\\ 2+3 & (-2)+1 & 0+3\end{bmatrix} = \begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$ ——— (1)

R.H.S = $A+(A+B)$

$= \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right) = \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1+1 & 2+(-1) & 3+1\\ 2+2 & 3+(-2) & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$

$= \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix} = \begin{bmatrix}1+2 & 2+1 & 3+4\\ 2+4 & 3+1 & 1+3\\ 1+4 & -1+0 & 0+3\end{bmatrix} = \begin{bmatrix}3 & 3 & 7\\ 6 & 4 & 4\\ 5 & -1 & 3\end{bmatrix}$ ——— (2)

From "1" and "2", it is proved that: $2A+B = A+(A+B)$

(ix) $A+(B-C) = (A-C)+B$

Solution: $A+(B-C) = (A-C)+B$

L.H.S = $A+(B-C)$

$= \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix} - \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right) = \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1-(-1) & -1-0 & 1-0\\ 2-0 & -2-(-2) & 2-3\\ 3-1 & 1-1 & 3-2\end{bmatrix}$

$= \begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} + \begin{bmatrix}0 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix} = \begin{bmatrix}1+0 & 2+(-1) & 3+1\\ 2+2 & 3+0 & 1+(-1)\\ 1+2 & -1+0 & 0+1\end{bmatrix} = \begin{bmatrix}1 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$ ——— (1)

R.H.S = $(A-C)+B$

$= \left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix} - \begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right) + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix} = \begin{bmatrix}1-(-1) & 2-0 & 3-0\\ 2-0 & 3-(-2) & 1-3\\ 1-1 & -1-1 & 0-2\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$

$\begin{bmatrix}0 & 2 & 3\end{bmatrix} \quad \begin{bmatrix}1 & -1 & 1\end{bmatrix} \quad \begin{bmatrix}0+1 & 2+(-1) & 3+1\end{bmatrix} \quad \begin{bmatrix}1 & 1 & 4\end{bmatrix}$

(x) $2A + 2B = 2(A + B)$

Solution: $2A + 2B = 2(A + B)$

L.H.S = $2A + 2B$

$$= \left(2 \times \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}\right) + \left(2 \times \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) = \begin{bmatrix}2\times1 & 2\times2 & 2\times3\\2\times2 & 2\times3 & 2\times1\\2\times1 & 2\times(-1) & 2\times0\end{bmatrix} + \begin{bmatrix}2\times1 & 2\times(-1) & 2\times1\\2\times2 & 2\times(-2) & 2\times2\\2\times3 & 2\times1 & 2\times3\end{bmatrix}$$

$$= \begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix} + \begin{bmatrix}2 & -2 & 2\\4 & -4 & 4\\6 & 2 & 6\end{bmatrix} = \begin{bmatrix}2+2 & 4+(-2) & 6+2\\4+4 & 6+(-4) & 2+4\\2+6 & -2+2 & 0+6\end{bmatrix} = \begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix} \quad \text{———— (1)}$$

R.H.S = $2(A + B)$

$$= 2 \times \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) = 2 \times \begin{bmatrix}1+1 & 2+(-1) & 3+1\\2+2 & 3+(-2) & 1+2\\1+3 & -1+1 & 0+3\end{bmatrix}$$

$$= 2 \times \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix} = \begin{bmatrix}2\times2 & 2\times1 & 2\times4\\2\times4 & 2\times1 & 2\times3\\2\times4 & 2\times0 & 2\times3\end{bmatrix} = \begin{bmatrix}4 & 2 & 8\\8 & 2 & 6\\8 & 0 & 6\end{bmatrix} \quad \text{———— (2)}$$

From "1" and "2", it is proved that: $2A + 2B = 2(A + B)$

Q8. If $A = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$, $B = \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}$, then verify that:

(i) $(A + B)^t = A^t + B^t$ (ii) $(A - B)^t = A^t - B^t$ (iii) $A + A^t$ is symmetric

(iv) $A - A^t$ is skew symmetric ; EX #1.3 Q8. (i , ii, iii , iv)

Solution: (i) $(A + B)^t = A^t + B^t$

L.H.S = $(A + B)^t = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t = \left(\begin{bmatrix}1+1 & 2+1\\0+2 & 1+0\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}2 & 3\\2 & 1\end{bmatrix}\right)^t = \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$ ———— (i)

R.H.S = $A^t + B^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t + \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1+1 & 0+2\\2+1 & 1+0\end{bmatrix} = \begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$ ———— (ii)

From (i) and (ii), it is proved that: $(A + B)^t = A^t + B^t$

(ii) $(A - B)^t = A^t - B^t$

Solution: $(A - B)^t = A^t - B^t$

L.H.S = $(A - B)^t = \left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}\right)^t = \begin{bmatrix}1-1 & 2-1\\0-2 & 1-0\end{bmatrix}^t$

$= \begin{bmatrix}0 & 1\\-2 & 1\end{bmatrix}^t = \begin{bmatrix}0 & -2\\1 & 1\end{bmatrix}$ ———— (i)

R.H.S = $A^t - B^t$

$= \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t - \begin{bmatrix}1 & 1\\2 & 0\end{bmatrix}^t = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\1 & 0\end{bmatrix}$

$= \begin{bmatrix}1-1 & 0-2\\2-1 & 2-0\end{bmatrix} = \begin{bmatrix}0 & -2\\1 & 2\end{bmatrix}$ ———— (ii)

From (i) and (ii), it is proved that: $(A - B)^t = A^t - B^t$

(iii) To prove $A + A^t$ is symmetric

Solution: $A + A^t$ is symmetric

$A + A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

$= \begin{bmatrix}1+1 & 2+0\\0+2 & 1+1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$ ———— (1)

Now we will take transpose of $A + A^t$

$(A + A^t)^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}^t = \begin{bmatrix}2 & 2\\2 & 2\end{bmatrix}$ ———— (2)

$$A - A^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}^t = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1-1 & 2-0\\0-2 & 1-1\end{bmatrix} = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix} \quad \text{---------- (i)}$$

Now take the transpose of (i), we have:

$$(A - A^t)^t = \begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}^t = \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix} = (-1)\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix}$$

$$= -\begin{bmatrix}0 & 2\\-2 & 0\end{bmatrix} \quad \text{-------- (ii)}$$

$$= -(A - A^t)$$

From (i) and (ii), it is obvious that: $A - A^t$ is skew symmetric

Q1. Which of the following product of matrices is conformable for multiplication

(i) $\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}-2\\3\end{bmatrix}$ (ii) $\begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$ (iii) $\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 2\\0 & -1\\-1 & -2\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\end{bmatrix}$ (v) $\begin{bmatrix}3 & 2 & 1\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 2\\-2 & 3\end{bmatrix}$; **EX #1.4 Q1.**

Solution:

Two matrices are conformable for multiplication if the numbers of columns of first matrix are equal to number of rows of second matrix

So, according to the definition:

(i) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(ii) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(iii) is not conformable for multiplication (because the first matrix has just one column and second matrix has two rows).

(iv) is conformable for multiplication (because the first matrix has two columns and second matrix has same number of rows).

(v) is conformable for multiplication (because the first matrix has three columns and second matrix has same number of rows).

Q3. Find the following products

(i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ (ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix}$ (iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$ (iv) $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

(v) $\begin{bmatrix}1 & 2\\-3 & 0\\6 & -1\end{bmatrix}\begin{bmatrix}4 & 5\\0 & -4\end{bmatrix}$; **EX #1.4 Q3.**

Solution: (i) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}1\times 4 + 2\times 0\end{bmatrix} = \begin{bmatrix}4+0\end{bmatrix} = \begin{bmatrix}4\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}4\end{bmatrix}$

Solution: (ii) $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix} = \begin{bmatrix}1\times 5 + 2\times(-4)\end{bmatrix} = \begin{bmatrix}5-8\end{bmatrix} = \begin{bmatrix}-3\end{bmatrix}$

So, $\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}5\\-4\end{bmatrix} = \begin{bmatrix}-3\end{bmatrix}$

(iii) $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $= \begin{bmatrix}(-3)\times 4 + 0\times 0\end{bmatrix} = \begin{bmatrix}-12\end{bmatrix}$; So, $\begin{bmatrix}-3 & 0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}-12\end{bmatrix}$

(iv) $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix}$

Solution: $= \begin{bmatrix}6\times 4 + 0\times 0\end{bmatrix} = \begin{bmatrix}24+0\end{bmatrix} = \begin{bmatrix}24\end{bmatrix}$; So, $\begin{bmatrix}6 & -0\end{bmatrix}\begin{bmatrix}4\\0\end{bmatrix} = \begin{bmatrix}24\end{bmatrix}$

Solution: $= \begin{bmatrix} 1\times4+2\times0 & 1\times5+2\times(-4) \\ -3\times4+0\times0 & -3\times5+0\times(-4) \\ 6\times4+(-1)\times0 & 6\times5+(-1)\times(-4) \end{bmatrix} = \begin{bmatrix} 4+0 & 5+(-8) \\ -12+0 & -15+0 \\ 24+0 & 30+4 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$; So, $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$

Q5. Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$. Verify whether

(i) AB = BA (ii) A (BC) = (AB) C (iii) A (B + C) = AB + AC

(iv) A (B − C) = AB − AC ; EX #1.4 Q5.

(i) AB=BA

Solution:

L.H.S = AB = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} -1\times1 & 3\times2 \\ 2\times(-3) & 0\times(-5) \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ -6 & 0 \end{bmatrix}$

R.H.S = BA = $\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1\times(-1) & 2\times3 \\ -3\times2 & -5\times0 \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ -6 & 0 \end{bmatrix}$

Therefore L.H.S = R.H.S ; AB = BA

(ii) A (BC) = (AB) C

Solution:

L.H.S = A(BC) = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1\times2+2\times1 & 1\times1+2\times3 \\ -3\times2+(-5)\times1 & -3\times1+(-5)\times3 \end{bmatrix}\right) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix} = \begin{bmatrix} -1\times4+3\times(-11) & -1\times7+3\times(-18) \\ 2\times4+0\times(-11) & 2\times7+0\times(-18) \end{bmatrix}$

$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix} = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$ --------- (i)

R.H.S = (AB) C

$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -1\times1+3\times(-3) & -1\times2+3\times(-5) \\ 2\times1+0\times(-3) & 2\times2+0\times(-5) \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -10\times2+(-17)\times1 & (-10)\times1+(-17)\times3 \\ 2\times2+4\times1 & 2\times1+4\times3 \end{bmatrix}$

$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix} = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix}$ ---------- (ii)

From (i) and (ii), it is obvious that: L.H.S = R.H.S ; A (BC) = (AB) C

(iii) A (B + C) = AB + AC

Solution: L.H.S = A (B + C) = $\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1+2 & 2+1 \\ -3+1 & -5+3 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 3 & 3 \\ -2 & -2 \end{bmatrix}$

$= \begin{bmatrix} -1\times3+3\times(-2) & -1\times3+3\times(-2) \\ 2\times3+0\times(-2) & 2\times3+0\times(-2) \end{bmatrix} = \begin{bmatrix} -3-6 & -3-6 \\ 6+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} -9 & -9 \\ 6 & 6 \end{bmatrix}$ --------- (i)

R.H.S = AB + AC

$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1\times1+3\times(-3) & -1\times2+3\times(-5) \\ 2\times1+0\times(-3) & 2\times2+0\times(-5) \end{bmatrix} + \begin{bmatrix} -1\times2+3\times1 & -1\times1+3\times3 \\ 2\times2+0\times1 & 2\times1+0\times3 \end{bmatrix}$

$= \begin{bmatrix} -1-9 & -2-15 \end{bmatrix} + \begin{bmatrix} -2+3 & -1+9 \end{bmatrix} = \begin{bmatrix} -10 & -17 \end{bmatrix} + \begin{bmatrix} 1 & 8 \end{bmatrix}$

(iv) $A(B-C) = AB - AC$

Solution: L.H.S $= A(B-C) = \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix} - \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix} \times \begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix} = \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix} \times \begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$

$= \begin{bmatrix}-1\times(-1)+3\times(-4) & -1\times 1+3\times(-8)\\ 2\times(-1)+0\times(-4) & 2\times 1+0\times(-8)\end{bmatrix}$

$= \begin{bmatrix}1-12 & -1-24\\ -2+0 & 2+0\end{bmatrix} = \begin{bmatrix}11 & -25\\ -2 & 2\end{bmatrix}$ -------- (i)

R.H.S $= AB - AC$

$= \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix} \times \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix} - \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix} \times \begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix} = \begin{bmatrix}-1\times 1+3\times(-3) & -1\times 2+3\times(-5)\\ 2\times 1+0\times(-3) & 2\times 2+0\times(-5)\end{bmatrix}$

$- \begin{bmatrix}-1\times 2+3\times 1 & -1\times 1+3\times 3\\ 2\times 2+0\times 1 & 2\times 1+0\times 3\end{bmatrix} = \begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix} - \begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$

$= \begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix} - \begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix} = \begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$

$= \begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$ ---------- (ii)

From (i) and (ii), hence proved: L.H.S = R.H.S ; $A(B-C) = AB - AC$

Q6. For the matrices

$A = \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B = \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$, $C = \begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that (i) $(AB)^t = B^tA^t$ (ii) $(BC)^t = C^tB^t$; EX #1.4 Q6.

(i) $(AB)^t = B^tA^t$

Solution: L.H.S $= (AB)^t = \left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix} \times \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}-1\times 1+3\times(-3) & -1\times 2+3\times(-5)\\ 2\times 1+0\times(-3) & 2\times 2+0\times(-5)\end{bmatrix}\right)^t$

$= \left(\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}\right)^t = \left(\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\right)^t = \begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$ ------------ (i)

R.H.S $= B^tA^t$

$= \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t \times \begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}^t = \begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix} \times \begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$

$= \begin{bmatrix}1\times(-1)+(-3)\times 3 & 1\times 2+(-3)\times 0\\ 2\times(-1)+(-5)\times 3 & 2\times 2+(-5)\times 0\end{bmatrix}$

$= \begin{bmatrix}-1-9 & 2+0\\ -2-15 & 4+0\end{bmatrix} = \begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$ ------------- (ii)

From (i) and (ii), it is proved that: L.H.S = R.H.S ; $(AB)^t = B^tA^t$

(ii) $(BC)^t = C^tB^t$

Solution: L.H.S $= (BC)^t = \left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix} \times \begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}\right)^t$

$= \begin{bmatrix}1\times(-2)+2\times 3 & 1\times 6+2\times(-9)\\ -3\times(-2)+(-5)\times 3 & -3\times 6+(-5)\times(-9)\end{bmatrix}^t$

$= \begin{bmatrix}-2+6 & 6-18\\ 6-15 & -18+45\end{bmatrix}^t = \begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t = \begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$ ---------- (i)

R.H.S $= C^tB^t$

$= \begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}^t \times \begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t = \begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix} \times \begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$

$= \begin{bmatrix}-2\times 1+3\times 2 & -2\times(-3)+3\times(-5)\\ 6\times 1+(-9)\times 2 & 6\times(-3)+(-9)\times(-5)\end{bmatrix}$

Q2. Find which of the following matrices are singular or non-singular?

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$; **EX #1.5 Q2. (i , ii)**

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

Solution: A matrix is said to be singular if its determinant is equal to zero. i.e. $|A| = 0$.

Determinant of matrix A is calculated as: $|A| = \det A = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = 3 \times 4 - 2 \times 6$

$|A| = 12 - 12 = 0$

As, determinant of A is equal to zero so, A is a singular matrix.

(ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

Solution: Determinant of matrix B is calculated as: $|B| = \det B = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = 4 \times 2 - 3 \times 1$

$|A| = 8 - 3 = 5 \neq 0$

As, determinant of B is not equal to zero so, B is a non singular matrix.

Q3. Find the multiplicative inverse (if it exists) of each.

(iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$ (iv) $D = \begin{bmatrix} 1/2 & 3/4 \\ 1 & 2 \end{bmatrix}$; **EX #1.5 Q3. (iii , iv)**

Solution: (iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

The multiplicative inverse of matrix C is calculated as: $C^{-1} = \frac{Adj\ C}{|C|}$

$Adj\ C = \begin{bmatrix} -9 & -6 \\ -3 & -2 \end{bmatrix}$

$|C| = \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix} = (-9) \times (-2) - (-3) \times (-6) = 18 - 18 = 0$

Since it is a singular matrix therefore solution is not possible

$C^{-1} = \frac{\begin{bmatrix} -9 & -6 \\ -3 & -2 \end{bmatrix}}{0} = \infty$; C^{-1} does not exist.

(iv) $D = \begin{bmatrix} 1/2 & 3/4 \\ 1 & 2 \end{bmatrix}$

Solution: The multiplicative inverse of matrix D is calculated as: $D^{-1} = \frac{Adj\ D}{|D|}$

$Adj\ D = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$

$|D| = \begin{vmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{vmatrix} = \frac{1}{2} \times 2 - 1 \times \frac{3}{4} = 1 - \frac{3}{4}$

$= \frac{4-3}{4} = \frac{1}{4} \neq 0$

Since it is a non-singular matrix therefore solution is possible

$D^{-1} = \frac{\begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}}{\frac{1}{4}} = \begin{bmatrix} 2 \times 4 & \frac{-3}{4} \times 4 \\ -1 \times 4 & \frac{1}{2} \times 4 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$

$D^{-1} = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$

Q4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$, then (i) $A(Adj\ A) = (Adj\ A)\ A = (\det A) I$

(i) $A(Adj\ A) = (Adj\ A)\ A = (det\ A)I$

Adj A = $\begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$

det A = $\begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 1 \times 6 - 4 \times 2 = 6 - 8 = -2$

Now, $A\,(Adj\ A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 1\times 6 + 2 \times (-4) & 1 \times (-2) + 2 \times 1 \\ 4 \times 6 + 6 \times (-4) & 4 \times (-2) + 6 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ --------- (i)

$(Adj\ A)\ A$

$= \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 6\times 1 + (-2) \times 4 & 6 \times 2 + (-2) \times 6 \\ -4 \times 1 + 1 \times 4 & -4 \times 2 + 1 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ --------- (ii)

$(det\ A)\ I$

$= (-2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$ --------- (iii)

From (i), (ii) and (iii), it is clear that: $A(Adj\ A) = (Adj\ A)\ A = (det\ A)I$ Hence proved:

(ii) $BB^{-1} = I = B^{-1}B$

Solution: As, $B^{-1} = \frac{Adj\ B}{\det B}$

Adj B = $\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$

det B = $\begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix} = 3 \times (-2) - 2 \times (-1)$

$= -6 + 2 = -4 \neq 0$

$B^{-1} = \frac{\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}}{-4} = \begin{bmatrix} -2/-4 & 1/-4 \\ -2/-4 & 3/-4 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & -3/4 \end{bmatrix}$

Now, BB^{-1}

$= \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \begin{bmatrix} 1/2 & -1/4 \\ 1/2 & -3/4 \end{bmatrix} = \begin{bmatrix} 3 \times \left(\frac{1}{2}\right) + (-1) \times \left(\frac{1}{2}\right) & 3 \times \left(-\frac{1}{4}\right) + (-1) \times \left(-\frac{3}{4}\right) \\ 2 \times \left(\frac{1}{2}\right) + (-2) \times \left(\frac{1}{2}\right) & 2 \times \left(-\frac{1}{4}\right) + (-2) \times \left(-\frac{3}{4}\right) \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} - \left(\frac{1}{2}\right) & -\frac{3}{4} + \frac{3}{4} \\ 1 - 1 & -\frac{1}{2} + \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ --------- (i)

Now $B^{-1}B$

$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \times 3 + (-\frac{1}{4}) \times 2 & \frac{1}{2} \times (-1) + \left(-\frac{1}{4}\right) \times (-2) \\ \frac{1}{2} \times 3 + (-\frac{3}{4}) \times 2 & \frac{1}{2} \times (-1) + \left(-\frac{3}{4}\right) \times (-2) \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{2} - \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \\ \frac{3}{2} - \frac{3}{2} & -\frac{1}{2} + \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{3-1}{2} & \frac{-1+1}{2} \\ \frac{3-3}{2} & \frac{-1+3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = I$ --------- (ii)

From (i) and (ii), it is clear that: $BB^{-1} = I = B^{-1}B$ Hence proved

Q1. Use matrices, if possible, to solve the following systems of linear equations by:
(i) the matrix inverse method
(ii) the Cramer's rule. ; EX #1.6 Q1. (i , iii, v , vii)

(i) $2x - 2y = 4$; $3x + 2y = 6$ (iii) $4x + 2y = 8$; $3x - y = -1$

(v) $3x - 2y = 4$; $-6x + 4y = 7$ (vii) $2x - 2y = 4$; $-5x - 2y = -10$

Solution:

Step 1 $\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\6\end{bmatrix}$

Step 2 The coefficient matrix M = $\begin{bmatrix}2 & -2\\3 & 2\end{bmatrix}$ is non-singular

Because; det M = $\begin{vmatrix}2 & -2\\3 & 2\end{vmatrix} = 2\times 2 - 3\times(-2) = 4+6 = 10 \neq 0$

Step 3 $\begin{bmatrix}x\\y\end{bmatrix} = M^{-1}\begin{bmatrix}4\\6\end{bmatrix} \Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{|M|}Adj\,M\begin{bmatrix}4\\6\end{bmatrix} = \frac{1}{10}\begin{bmatrix}2 & 2\\-3 & 2\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}$

$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{10}\begin{bmatrix}2\times4+2\times6\\-3\times4+2\times6\end{bmatrix} = \frac{1}{10}\begin{bmatrix}8+12\\-12+12\end{bmatrix} = \frac{1}{10}\begin{bmatrix}20\\0\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

$\Rightarrow \quad x = 2, \quad y = 0$

(iii) $4x + 2y = 8 \quad ; \quad 3x - y = -1$

Solution:

Step 1 $\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}8\\-1\end{bmatrix}$

Step 2 The coefficient matrix M = $\begin{bmatrix}4 & 2\\3 & -1\end{bmatrix}$ is non-singular

because ; det M = $\begin{vmatrix}4 & 2\\3 & -1\end{vmatrix} = 4\times(-1) - 2\times 3 = -4-6 = -10 \neq 0$

Step 3

$\begin{bmatrix}x\\y\end{bmatrix} = M^{-1}\begin{bmatrix}8\\-1\end{bmatrix}$

$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{|M|}Adj\,M\begin{bmatrix}8\\-1\end{bmatrix} = -\frac{1}{10}\begin{bmatrix}-1 & -2\\-3 & 4\end{bmatrix}\begin{bmatrix}8\\-1\end{bmatrix} = \frac{1}{10}\begin{bmatrix}(-1)\times8+(-2)\times(-1)\\-3\times8+4\times(-1)\end{bmatrix}$

$= -\frac{1}{10}\begin{bmatrix}-8+2\\-24+-4\end{bmatrix} = -\frac{1}{10}\begin{bmatrix}-6\\-28\end{bmatrix} = \begin{bmatrix}\frac{3}{5}\\\frac{14}{5}\end{bmatrix}$

$\Rightarrow \quad x = \frac{3}{5}, \quad y = \frac{14}{5}$

(v) $3x - 2y = 4 \quad ; \quad -6x + 4y = 7$

Solution:

Step 1 $\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\7\end{bmatrix}$

Step 2 The coefficient matrix M = $\begin{bmatrix}3 & -2\\-6 & 4\end{bmatrix}$ is singular because

det M = $\begin{vmatrix}3 & -2\\-6 & 4\end{vmatrix} = 3\times 4 - (-2)\times(-6) = 12 - 12 = 0$

So, M is a singular matrix. Hence the system of linear equations has no solution.

(vii) $2x - 2y = 4 \quad ; \quad -5x - 2y = -10$

Solution:

Step 1 $\begin{bmatrix}2 & 2\\-5 & -2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\-10\end{bmatrix}$

Step 2

The coefficient matrix M = $\begin{bmatrix}2 & 2\\-5 & -2\end{bmatrix}$ is non-singular

because ; det M = $\begin{vmatrix}2 & 2\\-5 & -2\end{vmatrix} = 2\times(-2) - 5\times 2 = -4 - 10 = -14 \neq 0$

Step 3

$\begin{bmatrix}x\\y\end{bmatrix} = M^{-1}\begin{bmatrix}4\\-10\end{bmatrix}$

$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{|M|}Adj\,M\begin{bmatrix}4\\-10\end{bmatrix} = \frac{1}{\ }\begin{bmatrix}-2 & -2\\ & \end{bmatrix}\begin{bmatrix}4\\ \end{bmatrix}$

$\Rightarrow \quad x = 2, \quad y = 0$

(ii) Solution By Cramer's Rule:

(i) $2x - 2y = 4 \quad ; \quad 3x + 2y = 6$

Solution: $\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} = 2 \times 2 - 3 \times (-2) = 4 + 6 = 10 \neq 0$

$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix} = 4 \times 2 - 6 \times (-2) = 8 + 12 = 20$

$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 2 \times 6 - 3 \times 4 = 12 - 12 = 0$

$x = \dfrac{|A_x|}{|A|} = \dfrac{20}{10} = 2$

$y = \dfrac{|A_y|}{|A|} = \dfrac{0}{10} = 0$

So, $x = 2$ and $y = 0$

(iii) $4x + 2y = 8 \quad ; \quad 3x - y = -1$

Solution: $\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 2 = -4 - 6 = -10 \neq 0$

$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix} = 8 \times (-1) - 2 \times (-1) = -8 + 2 = -6$

$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix} = 4 \times (-1) - 3 \times 8 = -4 - 24 = -28$

$x = \dfrac{|A_x|}{|A|} = \dfrac{-6}{-10} = \dfrac{3}{5}$

$y = \dfrac{|A_y|}{|A|} = \dfrac{-28}{-10} = \dfrac{14}{5}$

So, $x = \dfrac{3}{5}$ and $y = \dfrac{14}{5}$

(v) $3x - 2y = 4 \quad ; \quad -6x + 4y = 7$

Solution: $\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix} = 3 \times 4 - (-6) \times (-2) = 12 - 12 = 0$

Since it is singular matrix therefore solution is not possible. So, x and y are not possible in this case.

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix} = 2\times(-2)-(-5)\times(-2) = -4-10 = -14 \neq 0$$

$$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$|A_x| = \begin{vmatrix} 4 & 2 \\ -10 & -2 \end{vmatrix} = 4\times(-2)-(-10)\times 2 = -8-20 = -28$$

$$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ -5 & -10 \end{vmatrix} = 2\times(-10)-(-5)\times 4 = -20+20 = 0$$

$$x = \frac{|A_x|}{|A|} = \frac{-28}{-14} = 2$$

$$y = \frac{|A_y|}{|A|} = \frac{0}{-14} = 0$$

So, $x = 2$ and $y = 0$

Q1. Select the correct answer in each of the following. Review EX #1 Q.1

(i) The order of matrix $[2 \quad 1]$ is.........
(a) 2-by-1 (b) 1-by-2 (c) 1-by-1 (d) 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called.........matrix.
(a) zero (b) unit (c) scalar (d) singular

(iii) Which is order of a square matrix..........
(a) 2-by-2 (b) 1-by-2 (c) 2-by-1 (d) 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is........
(a) 3-by-2 (b) 2-by-3 (c) 1-by-3 (d) 3-by-1

(v) Ad joint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is...........
(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $[x \quad y]\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is.........
(a) $[2x+y]$ (b) $[x-2y]$ (c) $[2x-y]$ (d) $[x+2y]$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix} = 0$, then x is equal to...a = ...
(a) 9 (b) -6 (c) 6 (d) -9

(viii) If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to........
(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

Answers:

(i) b	(ii) c	(iii) a	(iv) b
(v) a	(vi) c	(vii) a	(viii) d

GUESS PAPER & MODEL PAPER # 02
BASED ON UNIT # 2 (Reduced Syllabus)
REAL AND COMPLEX NUMBERS

Unit 2	Real and Complex Numbers
Exercise 2.1	Q3; Q4(I, ii, iii); Q6(i, ii)
Exercise 2.2	Q1; Q3
Exercise 2.3	Q1(i, ii); Q3(i, ii)
Exercise 2.4	Q1(i, iv); Q2; Q3(i, ii)
Exercise 2.5	Q1(i, ii, iv); Q2(i, ii, iii); Q3(iv, v); Q4
Exercise 2.6	Q1; Q2(ii, iv); Q3(ii, iv); Q4(i, iv, v); Q5(ii, iii); Q6(i, iii, iv, v); Q7(i, ii)
Review Ex 2	Q1; Q2; Q5; Q7

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) $(27x^{-1})^{-2/3}$ ....

(A) $\frac{\sqrt[3]{x^2}}{9}$ (B) $\frac{\sqrt{x^3}}{9}$ (C) $\frac{\sqrt[3]{x^2}}{8}$ (D) $\frac{\sqrt{x^3}}{8}$

(ii) Write $\sqrt[7]{x}$ in exponential form........

(A) x (B) x^7 (C) $x^{1/7}$ (D) $x^{7/2}$

(iii) Write $4^{2/3}$ with radical sign.........

(A) $\sqrt[3]{4^2}$ (B) $\sqrt{4^3}$ (C) $\sqrt[3]{4^3}$ (D) $\sqrt{4^6}$

(iv) In $\sqrt[3]{35}$ the radicand is........

(A) 3 (B) $\frac{1}{3}$ (C) 35 (D) none of these

(v) $\left(\frac{25}{16}\right)^{-1/2}$ = ...

5 4 5 4

(vii) The value of i^9 is........
(A) 1 (B) -1 (C) i (D) $-i$

(viii) Every real number is........
(A) a positive integer (B) a rational number
(C) a negative integer (D) a complex number

(ix) Real part of $2ab(i+i^2)$ is.........
(A) $2ab$ (B) $-2ab$ (C) $2abi$ (D) $-2abi$

(x) Imaginary part of $-i(3i+2)$ is.....
(A) -2 (B) 2 (C) 3 (D) -3

(xi) Which of the following sets have the closure property w.r.t. additi . ...
(A) {0} (B) {0,-1} (C) {0 ' (D) $\{1, \sqrt{2}, 5\}$

(xii) Name the property of real numbers used in $\left(-\frac{\sqrt{5}}{2}\right) \times 1 = -\frac{\sqrt{5}}{2}$ ....
(A) additive identity (B) additive inverse (C) multiplicative identity (L multiplicative inverse

(xiii) If $z < 0$ then $x < y \Rightarrow$......
(A) $xz < yz$ (B) $xz > yz$ (C) $xz = yz$ ( ) none of these

(xiv) If $a, b \in R$ then only one of $a = b$ or $a < b$ or $a > b$ holds is called ......
(A) trichotomy property (B) tr sitive property
(C) additive property (D) multiplicative property

(xv) A non-terminating, non-recurring decimal represents:
(A) a natural number (B) a rational number (C) an irrational number (D) a prime number

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–P if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. (9 × 4 = 36)

(i) Simplify and write your answer in the form $a+bi$. $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$ EX #2.6 Q.4 ; ;(iv)

(ii) Solve the following equations for real x and y. $(2-3i)(x+yi) = 4+i$; EX #2.6 Q.7 ; ;(i)

(iii) Simplify $\left(\frac{a^p}{a^q}\right)^{p+q} \cdot \left(\frac{a^q}{a^r}\right)^{q+r} \div 5(a^p \cdot a^r)^{p-r}$, $a \neq 0$; Review EX #2 Q.5

(iv) Simplify and write your answer in the form $a+bi$. $\left(\frac{1+i}{1-i}\right)^2$; EX #2.6 Q.4 ;(v)

(v) Express each complex number in the standard form $a+bi$, where a and b are real numbers. $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$; EX #2.6 Q.2 ; ;(iv)

(vi) Find the value of x and y if $x + iy + 1 = 4 - 3i$. ; EX #2.5 Q.4

(vii) Use laws of exponents to simplify: $\sqrt{\frac{(216)^{2/3} \times (25)^{1/2}}{(0.04)^{-1/2}}}$; EX #2.4 Q.3 ; ;(ii)

(viii) Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1$. ; EX #2.4 Q.2

(ix) Use laws of exponents to simplify: $\frac{(81)^n \cdot 3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$; EX #2.4 Q.1 ; ;(iv)

(xi) Simplify $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$

(xii) Simplify $\frac{(2)^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{-1}{3}} \times (9)^{\frac{1}{4}}}$; EX #2.4 Q.3 ; ;(i)

(xiii) Use laws of exponents to simplify: $\frac{(243)^{-2/3}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$; EX #2.4 Q.1 ; ;(i)

(xiv) Simplify and write your answer in the form $a + bi$. $\frac{-2}{1+i}$; EX #2.6 Q.4 ; ;(i)

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 One angle of a parallelogram is 130°. Find the measures of its remaining angles. EX #11.1 ; Q.1

Q.4 Show whether or not the points with vertices $(-1,1)$, $(5,4)$, $(2,-2)$ and $(-4,1)$ from a square. ; EX #9.2 ; Q.2

Q.5 Prove that if two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram. ; Theorem # 11.1.2

Q.6 Any point on the right bisector of a line segment is equidistant from its end points. ; Theorem # 12.1.1

Q.7 Construct the Δ ABC. Draw the bisectors of their angles and verify their concurrency. $m\overline{AB} = 3.6$ cm, $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 5.2$ cm. ; EX #17.2 Q.1;(iii)

SOLUTION OF GUESS PAPER & MODEL PAPER # 2
(Reduced Syllabus)

SECTION- A (MCQs)

i. A	ii. C	iii. A	iv. C	v. B	vi. C
vii. C	viii. D	ix. B	x. A	xi. A	xii. C
xiii. B	xiv. A	xv. C			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Simplify and write your answer in the form $a + bi$. $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$ EX #2.6 Q.4 ; ;(iv)

Solution:= $\frac{2-6i-4-i}{3+i} = \frac{-2-7i}{3+i} = \frac{-2-7i}{3+i} \times \frac{3-i}{3-i} = \frac{(-2-7i)(3-i)}{9-i^2}$

$= \frac{-6+2i-21i+7i^2}{9+1} = \frac{-6-19i-7}{10}$; $(\because i^2 = -1)$

$= \frac{-13-19i}{10} = -\frac{13}{10} - \frac{19}{10}i$

(ii) Solve the following equations for real x and y. $(2-3i)(x+yi) = 4+i$; EX #2.6 Q.7 ; ;(i)

Solution: $(2-3i)(x+yi) = 4+i \Rightarrow 2x + 2yi - 3xi - 3yi^2 = 4 + i$

$2x - 3y(-1) - 3xi + 2yi = 4 + i \Rightarrow (2x + 3y) + (2y - 3x)i = 4 + i$

Now multiplying eq. (i) by 3 and eq. (ii) by 2

$6x+9y=12$ .... (iii)

$-6x+4y=2$ .... (iv)

Adding eq. (iii) and eq. (iv)

$$\begin{array}{r} 6x+9y=12 \\ \underline{-6x+4y=2} \\ 13y=14 \end{array} \quad \Rightarrow \quad y=\frac{14}{13}$$

Put $y=\frac{14}{13}$ in eq. (i) $\quad 2x+3y=4$

$2x+3\left(\frac{14}{13}\right)=4 \;\Rightarrow\; 2x+\frac{42}{13}=4 \;\Rightarrow\; 2x=4-\frac{42}{13}=\frac{52-42}{13}=\frac{10}{13}$

$x=\frac{10}{13}\times\frac{1}{2}=\frac{5}{13}$; Hence $x=\frac{5}{13}$ and $y=\frac{14}{13}$

(iii) Simplify $\left(\frac{a^p}{a^q}\right)^{p+q}\cdot\left(\frac{a^q}{a^r}\right)^{q+r}\div 5(a^p\cdot a^r)^{p-r}$, $\quad a\neq 0$; Review EX #2 Q.5

Solution: $\left(\frac{a^p}{a^q}\right)^{p+q}\cdot\left(\frac{a^q}{a^r}\right)^{q+r}\div 5(a^p\cdot a^r)^{p-r}$

$= \left(\frac{a^p}{a^q}\right)^{p+q}\cdot\left(\frac{a^q}{a^r}\right)^{q+r}\div 5(a^p\cdot a^r)^{p-r}$

$= (a^p\cdot a^{-q})^{p+q}\cdot(a^q\cdot a^{-r})^{q+r}\div 5(a^{p+r})^{p-r}$

$= (a^{p-q})^{p+q}\cdot(a^{q-r})^{q+r}\div 5(a^{p+r})^{p-r}$

$= a^{p^2-q^2}\cdot a^{q^2-r^2}\div 5a^{p^2-r^2} = \frac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}} = \frac{a^{p^2-r^2-p^2+r^2}}{5}$

$= \frac{a^0}{5}=\frac{1}{5}$; $(\because a^0=1)$

(iv) Simplify and write your answer in the form $a+bi$. $\left(\frac{1+i}{1-i}\right)^2$; EX #2.6 Q.4 ;(v)

Solution: $= \left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^2 = \left(\frac{(1+i)^2}{1-i^2}\right)^2 = \left(\frac{1+2i+i^2}{1+1}\right)^2 = \left(\frac{1+2i-1}{2}\right)^2$

$= \left(\frac{2i}{2}\right)^2 = i^2 = -1$; $(\because i^2=-1)$

(v) Express each complex number in the standard form $a+bi$, where a and b are real numbers.
$2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$; EX #2.6 Q.2 ; ;(iv)

Solution: $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$

- By separating real and imaginary parts, we get

$= 2(-1)+6i\cdot i^2+3(i^2)^8-6i^{18}\cdot i+4i^{24}\cdot i$

$= -2+6i(-1)+3(1)-6(i^2)^9 i+4(i^2)^{22} i$

$= -2-6i+3-6i(-1)+4i$; $(\because i^2=-1)$

$= -2-6i+3+6i+4i = 1+4i$

(vi) Find the value of x and y if $x+iy+1=4-3i$. ; EX #2.5 Q.4

Solution: $x+iy+1 = 4-3i \Rightarrow (x+1)+iy = 4-3i$

By comparing real and imaginary parts, we get

(vii) Use laws of exponents to simplify: $\sqrt{\dfrac{(216)^{2/3} \times (25)^{1/2}}{(0.04)^{-1/2}}}$; EX #2.4 Q.3 ; ;(ii)

Solution: $\sqrt{\dfrac{(216)^{2/3} \times (25)^{1/2}}{(0.04)^{-1/2}}} = \sqrt{\dfrac{(2^3 \,.\, 3^3)^{2/3} \quad (5^2)^{1/2}}{\left(\frac{4}{100}\right)^{-1/2}}}$

$= \sqrt{2^2 \,.\, 3^2 \,.\, 5 \,.\, \left(\frac{4}{100}\right)^{1/2}} = \sqrt{2^2 \,.\, 3^2 \,.\, 5 \,.\, \left(\frac{2^2}{10^2}\right)^{1/2}}$

$= \sqrt{2^2 \,.\, 3^2 \,.\, 5 \,.\, \left(\frac{2}{10}\right)^{2\times\frac{1}{2}}} = \sqrt{2^2 \,.\, 3^2 \,.\, 5 \,.\, \frac{2}{10}}$

$= \sqrt{2^2 \,.\, 3^2} = 2 \,.\, 3 = 6$

(viii) Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = 1.$; EX #2.4 Q.2

Solution: L.H.S $= \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$

$= \left(x^a . x^{-b}\right)^{a+b} \times \left(x^b . x^{-c}\right)^{b+c} \times \left(x^c . x^{-a}\right)^{c+a}$

$= \left(x^{a-b}\right)^{a+b} \times \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a} = x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

(ix) Use laws of exponents to simplify: $\dfrac{(81)^n \,.\, 3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$; EX #2.4 Q.1 ; ;(iv)

Solution: $\dfrac{(81)^n \,.\, 3^5-(3)^{4n-1}(243)}{(9^{2n})(3^3)}$

$= \dfrac{(3^4)^n \,.\, 3^5-(3)^{4n-1} \,.\, 3^5}{(3^2)^{2n} \; (3^3)} = \dfrac{3^{4n} \,.\, 3^5-3^{4n-1} \,.\, 3^5}{3^{4n} \,.\, 3^3}$

$= \dfrac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}} = \dfrac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$

$= \dfrac{3^{4n+4}\,(3-1)}{3^{4n+3}} = 3^{4n+4-4n-3} \,.\, (2) = (3) \times (2) = 6$

(x) Solve the following equations for real x and y.
$(3-2i)(x+yi) = 2(x-2yi)+2i-1$; EX #2.6 Q.7 ; ;(ii)

Solution: $(3-2i)(x+yi) = 2(x-2yi)+2i-1$

$3x+3yi-2xi-2yi^2 = 2x-4yi+2i-1 \Rightarrow \quad 3x-2y(-1)-3yi+2xi = 2x-1-4yi+2i$

$(3x+2y)+(3y-2x)i = (2x-1)+(2-4y)i$

$3y - 2x = 2 - 4y$

or $3y - 2x + 4y = 2$

or $7y - 2x = 2$ .... (ii)

Multiplying eq. (i) by 2 and add in eq. (ii)

$$\begin{array}{r} 2x + 4y = -2 \\ \underline{-2x + 7y = 2} \\ 11y = 0 \end{array} \Rightarrow \quad y = 0$$

Put $y = 0$ in eq. (i)

$x + 2y = -1$

$x + 2(0) = -1 \quad \Rightarrow \quad x = -1$

Hence $x = -1$ and $y = 0$

(xi) Simplify $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$

Solution: $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$

$= \sqrt[3]{a^l \,.\, a^{-m}} \times \sqrt[3]{a^m \,.\, a^{-n}} \times \sqrt[3]{a^n \,.\, a^{-l}}$

$= \sqrt[3]{a^{l-m}} \times \sqrt[3]{a^{m-n}} \times \sqrt[3]{a^{n-l}} = \sqrt[3]{a^{l-m} \,.\, a^{m-n} \,.\, a^{n-l}}$

$= \sqrt[3]{a^{l-m+m-n+n-l}} = \sqrt[3]{a^0}$; $(\because \ a^0 = 1)$

$= \sqrt[3]{1} = (1)^{1/3} = 1$

(xii) Simplify $\dfrac{(2)^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{\frac{-1}{3}} \times (9)^{\frac{1}{4}}}$; EX #2.4 Q.3 ; ;(i)

Solution: $= \dfrac{2^{1/3} \times (3^3)^{1/3} \times (3 \,.\, 5 \,.\, 2^2)^{1/2}}{(2 \,.\, 2 \,.\, 2 \,.\, 3 \,.\, 3 \,.\, 5)^{1/2} \times (2^2)^{-1/3} \times (3^2)^{1/4}}$

$= \dfrac{2^{1/3} \,.\, 3^{3\times1/3} \,.\, 3^{1/2} \,.\, 5^{1/2} \,.\, 2^{2\times1/2}}{(2^2 \,.\, 3^2 \,.\, 5)^{1/2} \,.\, 2^{-2/3} \,.\, 3^{2/4}}$

$= \dfrac{2^{1/3} \,.\, 3 \,.\, 3^{1/2} \,.\, 5^{1/2} \,.\, 2}{(2^2 \,.\, 3^2 \,.\, 5)^{1/2} \,.\, 2^{-2/3} \,.\, 3^{2/4}} = \dfrac{2^{1/3} \,.\, 3^{1/2} \,.\, 5^{1/2} \,.\, 3 \,.\, 2}{2 \,.\, 3 \,.\, 5^{1/2} \,.\, 2^{-2/3} \,.\, 3^{1/2}}$

$= \dfrac{2^{\frac{1}{3}+\frac{2}{3}} \,.\, 3^{\frac{1}{2}} \,.\, 5^{\frac{1}{2}} \,.\, 3 \,.\, 2}{5^{\frac{1}{2}} \,.\, 3^{\frac{1}{2}} \,.\, 2 \,.\, 3} = 2^{3/3} = 2^1 = 2$

(xiii) Use laws of exponents to simplify: $\dfrac{(243)^{-2/3}\,(32)^{-1/5}}{\sqrt{(196)^{-1}}}$; EX #2.4 Q.1 ; ;(i)

$$= \frac{(243)^{-2/3}(32)^{-1/5}}{(196)^{-\frac{1}{2}}} = \left(\frac{1}{243}\right)^{2/3} \times \left(\frac{1}{32}\right)^{1/5} \times (196)^{1/2}$$
$$= \left(\frac{1}{3^5}\right)^{2/3} \times \left(\frac{1}{2^5}\right)^{1/5} \times (4 \times 49)^{1/2}$$
$$= \frac{1}{3^{10/3}} \times \frac{1}{2^{5/5}} \times (2^2)^{1/2} \times (7^2)^{1/2}$$
$$= \frac{1}{3^{1/3} \times 3^{9/3}} \times \frac{1}{2} \times 2 \times 7 \quad ; \quad \left(\because \frac{1}{3^{10/3}} = \frac{1}{3^{1/3} \times 3^{9/3}}\right)$$
$$= \frac{1}{3^3 \times 3^{1/3}} \times 7 = \frac{7}{3^3 \times \sqrt[3]{3}} = \frac{7}{27\left(\sqrt[3]{3}\right)}$$

(xiv) Simplify and write your answer in the form $a + bi$. $\frac{-2}{1+i}$; EX #2.6 Q.4 ; ;(i)

Solution: $\frac{-2}{1+i}$

$$= \frac{-2}{1+i} \times \frac{1-i}{1-i} = \frac{-2(1-i)}{1-i^2} = \frac{-2+2i}{1+1} = \frac{-2+2i}{2} \quad ; \quad (\because i^2 = -1)$$
$$= -1 + i$$

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 One angle of a parallelogram is 130°. Find the measures of its remaining angles.
EX #11.1 ; Q.1

Solution:

In parallelogram ABCD

m∠B = 130°

∠D ≅ ∠B

Opposite angles of a parallelogram

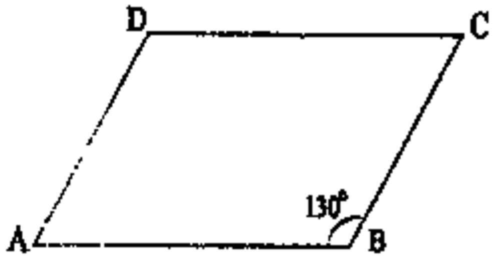
∴ m∠D = m∠B = 130°

m∠B + m∠A = 180°

130° + m∠A = 180°

∴ m∠A = 180° − 130° = 50°

m∠C = m∠A = 50° ; So unknown angles of parallelogram are 130°, 50°



Q.4 Show whether or not the points with vertices $(-1,1)$, $(5,4)$, $(2,-2)$ and $(-4,1)$ from a square. ; EX #9.2 ; Q.2

Solution: Let the points be A(-1,1), B(5,4), C(2,2) and D (-4,1)

Distance formula $= d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

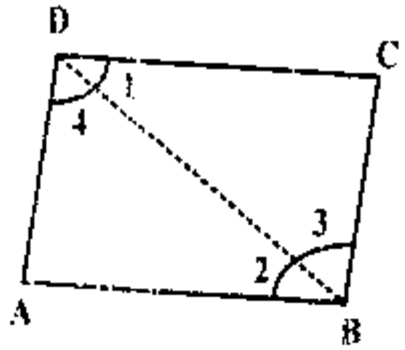
$|AB| = \sqrt{(5+1)^2 + (4-1)^2} = \sqrt{36+9} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$|BC| = \sqrt{(5-2)^2 + (4+2)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$|CD| = \sqrt{(2+4)^2 + (-2-1)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$|DA| = \sqrt{(-1+4)^2 + (1-1)^2} = \sqrt{(3)^2 + 0^2} = 3$

Q.5 Prove that if two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram. ; Theorem # 11.1.2



Solution:

Given:

In a quadrilateral ABCD

$\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$

To Prove:

ABCD is a parallelogram

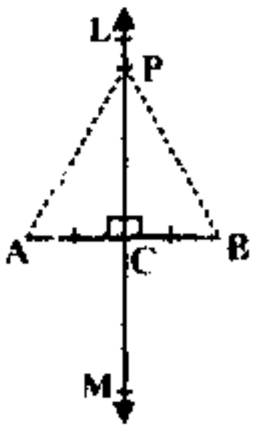
Construction:

Join the point B to D and in the figure name the angles as: $\angle 1, \angle 2, \angle 3,$ and $\angle 4$

Proof:

Statements	Reasons
$\angle 1 \cong \angle 2$	Alternate angles
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\angle 2 \cong \angle 1$	Alternate Angles
$\overline{BD} \cong \overline{BD}$	Common
$\therefore \Delta ABD \cong \Delta CDB$	S.A.S postulate
and $\angle 4 \cong \angle 3$ .... (i)	corresponding angles of congruent triangles
$\therefore \overline{AD} \parallel \overline{BC}$ .... (ii)	From (i)
and $\overline{AD} \parallel \overline{DC}$ .... (iii)	Given
Thus ABCD is a parallelogram	From (ii) and (iii)

Q.6 Any point on the right bisector of a line segment is equidistant from its end points.
Theorem # 12.1.1



Solution:

Given:

A line $\overleftrightarrow{LM}$ intersects the line segment AB at point C such that $\overleftrightarrow{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$.

To Prove:

$\overline{PA} \cong \overline{PB}$

Construction:

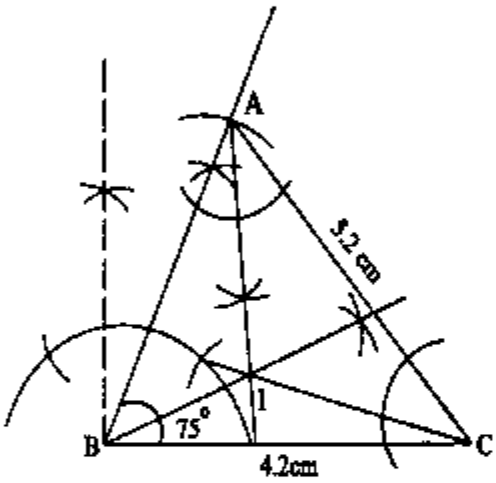
Take a point P on $\overleftrightarrow{LM}$. Join P to the points A and B.

Proof:

Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	Given $(\overline{PC} \perp \overline{AB})$
$\overline{PC} \cong \overline{PC}$	Common
$\Delta ACP \cong \Delta BCP$	S.A.S. Postulate
$\overline{PA} \cong \overline{PB}$	Corresponding sides of congruent triangles

Q.7 Construct the Δ ABC. Draw the bisectors of their angles and verify their concurrency.

Construction:



(i) Take $m\overline{BC} = 4.2$ cm.
(ii) With B as centre and radius $m\overline{BA} = 3.6$ cm draw an arc.
(iii) With C as centre and radius $m\overline{CA} = 5.2$ cm draw an arc.
(iv) Join $\overline{BA}$ and $\overline{CA}$ to complete the ΔABC.
(v) Draw bisectors of $\angle B$ and $\angle C$ meeting each other at the point I.
(vi) Now draw the bisector of the third $\angle A$.
(vii) We observe that the third angle bisector also passes through the point I.
(viii) Hence the angle bisectors of the ΔABC are concurrent at I.

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q3. Which of the following statements are true and which are false?

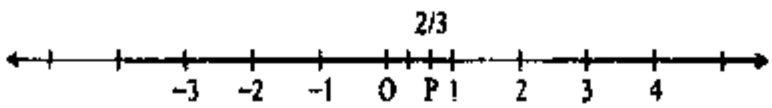
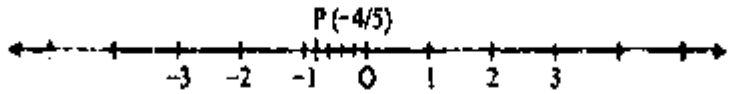
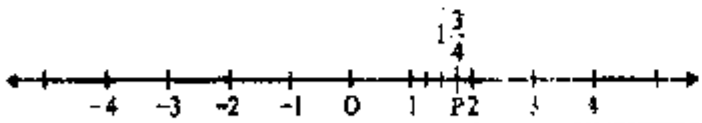
(i) $\frac{2}{3}$ **is an irrational number.** **(ii)** π **is an irrational number.**

(iii) $\frac{1}{9}$ **is a terminating fraction.** **(iv)** $\frac{3}{4}$ **is a terminating fraction.**

(v) $\frac{4}{5}$ **is a recurring fraction.** **; EX #2.1 Q.3**

Solution: (i) False (ii) True (iii) False (iv) True (v) False

4. Represent the following numbers on the number line. ; EX #2.1 Q.4 (i, ii, iii)

(i) $\frac{2}{3}$ **Solution:**	
(ii) $-\frac{4}{5}$ **Solution:**	
(iii) $1\frac{3}{4}$ **Solution:**	

Q6. Express the following recurring decimals as the rational number $\frac{p}{q}$ where p, q are integers and $q \neq 0$. (i) $0.\overline{5}$ (ii) $0.\overline{13}$ **; EX #2.1 Q.6 (i, ii)**

(i) $0.\overline{5}$

Solution: Let $x = 0.\overline{5}$

or $x = 0.5555\ldots\ldots$ .... (i)

Since we have only one digit i.e., 3 repeating indefinitely therefore multiplying both sides by 10

$10x = 5.555\ldots\ldots$ .... (ii)

(ii) $0.\overline{13}$

Solution: Let $x = 0.\overline{13}$

or $x = 0.131313\ldots\ldots$ .... (i)

Since we have only two digit i.e., 13 repeating indefinitely therefore multiplying both sides by 100

$100x = 13.131313\ldots\ldots$ .... (ii)

Subtracting (i) from (ii), we get

$100x - x = (13.1313\ldots\ldots) - (0.1313\ldots\ldots)$

$99x = 13.000$

Hence $x = \frac{13}{99}$

Q1. Identify the property used in the following

(i) $a + b = b + a$ (ii) $(ab)c = a(bc)$ (iii) $7 \times 1 = 7$ (iv) $x > y$ or $x = y$ or $x < y$

(v) $ab = ba$ (vi) $a + c = b + c \Rightarrow a = b$ (vii) $5 + (-5) = 0$ (viii) $7 \times \frac{1}{7} = 1$

(ix) $a > b \Rightarrow ac > bc \ (c > 0)$; EX #2.2 Q.1

Solution:
(i) Commutative Property w.r.t Addition
(ii) Associative Property w.r.t. Multiplication
(iii) Multiplicative identity
(iv) Trichotomy Property
(v) Commutative Property w.r.t Multiplication
(vi) Cancellation Property of Addition
(vii) Additive Inverse
(viii) Multiplicative Inverse
(ix) Multiplicative Property

Q3. Give the name of property used in the following.

(i) $\sqrt{24} + 0 = \sqrt{24}$ (ii) $-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$ (iii) $\pi + (-\pi) = 0$

(iv) $\sqrt{3}.\sqrt{3}$ is a real number (v) $\left(\frac{-5}{8}\right)\left(\frac{-8}{5}\right) = 1$; EX #2.2 Q.3

Solution:

(i) Additive Identity (ii) Distributive Property w.r.t. Multiplication

(iii) Additive Inverse (iv) Closure Property (v) Multiplicative Inverse

Q1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify. ; EX #2.3 Q.1 ;(i , ii)

Solution: (i) $\sqrt[3]{-64} = (-64)^{1/3}$

(ii) $2^{3/5} = (2^3)^{1/5} = \sqrt[5]{2^3}$

Q3. Simplify the following radical expressions. ; EX #2.3 Q.3 ;(i , ii)

(i) $\sqrt[3]{-125}$

Solution: $= \sqrt[3]{(-5)^3} = (-5)^{3\times\frac{1}{3}} = -5$

(ii) $\sqrt[4]{32}$

Solution: $= \sqrt[4]{2^5} = \sqrt[4]{2.\,2^4}$

$= \sqrt[4]{2}.\sqrt[4]{2^4} = \sqrt[4]{2}.(2)^{4\times\frac{1}{4}} = \sqrt[4]{2}.2 = 2\sqrt[4]{2}$

Q1. Evaluate ; EX #2.5 Q.1 ; ;(i , ii , iv)

Solution: (i) i^7

$= i^6 \times i = (i^2)^3 \times i$; $(\because i^2 = -1)$

$= (-1)^3 \times i = (-1) \times -i = -i$

Solution: (ii) i^{50}

$= \quad (-1)^4 \quad = \quad 1$

Q2. Write the conjugate of the following numbers. ; EX #2.5 Q.2 ; ;(i , ii , iii)

(i) $2+3i$

Solution: Let $z = 2+3i$; Then $\bar{z} = 2-3i$

(ii) $3-5i$

Solution: Let $z = 3-5i$; Then $\bar{z} = 3+5i$

(iii) $-i$

Solution: Let $z = -i$; Then $\bar{z} = i$

Q3. Write the real and imaginary part of the following numbers. ; EX #2.5 Q.3 ; ;(iv , v)

Solution: (iv) $-2-2i$ Re(z) $= -2$ Im(z) $= -2$

(v) $-3i$ Re(z) $= 0$ Im(z) $= -3$

Q1. Identify the following statements as true or false. EX #2.6 Q.1

(i) $\sqrt{-3}\sqrt{-3}=3$ (ii) $i^{73}=-i$ (iii) $i^{10}=-i$

(iv) Complex conjugate of $(-6i+i^2)$ is $(-1+6i)$

(v) Difference of a complex number $z=a+bi$ and its conjugate is a real number.

(vi) If $(a-1)-(b+3)i=5+8i$, then $a=6$ and $b=-11$

(vii) Product of a complex number and its conjugate is always a non-negative real number.

Solution:

(i) False (ii) False (iii) True (iv) True (v) False (vi) True (vii) True

Q2. Express each complex number in the standard form $a+bi$, where a and b are real numbers. ; EX #2.6 Q.2 ; ;(ii)

Solution: (ii) $2(5+4i)-3(7+4i)$

Solution: By separating real and imaginary parts, we get = $10+8i-21-12i = -11-4i$

Q3. Simplify and write your answer in the form $a+bi$. EX #2.6 Q.3 ; ;(ii , iv)

Solution: (ii) $(2-\sqrt{-4})(3-\sqrt{-4})$

Solution:= $(2-2i)(3-2i)$

$= 6-4i-6i+4i^2 = 6-10i+4(-1) = 6-10i-4$; $(\because i^2=-1)$

$= 2-10i$

(iv) $(2-3i)\overline{(3-2i)}$

Solution:= $(2-3i)(3+2i) = 6+4i-9i-6i^2$

$= 6-5i-6(-1)$; $(\because i^2=-1)$

$= 6+6-5i = 12-5i$

Q5. Calculate (a) $\bar{z}$ (b) $z+\bar{z}$ (c) $z-\bar{z}$ (d) $z\bar{z}$ for each of the following

(ii) $z=2+i$ (iii) $z=\frac{1+i}{1-i}$; EX #2.6 Q.5 ; ;(ii , iii)

Solution:

(ii) $z=2+i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=2+i+2-i=4$

(c) $z-\bar{z}=2+i-2+i=2i$

(d) $z\bar{z}=(2+i)(2-i)=4-i^2=4-(-1)=4+1=5$

(iii) $z=\frac{1+i}{1-i}$

$z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{1-i^2}=\frac{1+2i+i^2}{1-(-1)}=\frac{1+2i-1}{1+1}=\frac{2i}{2}=0+i$ (a) $\bar{z}=0-i=-i$

Q6. If $z = 2 + 3i$ and $w = 5 - 4i$, show that (i) $\overline{z+w} = \bar{z} + \bar{w}$ (iii) $\overline{zw} = \bar{z}\,\bar{w}$
(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$ where $w \neq 0$. (v) $\frac{1}{2}(z + \bar{z})$ is the real part of z. ;
EX #2.6 Q.6 ; ;(i , iii , iv , v)

Solution: $z = 2 + 3i \Rightarrow \bar{z} = 2 - 3i$
$w = 5 - 4i \Rightarrow \bar{w} = 5 + 4i$

(i) $\overline{z+w} = \bar{z} + \bar{w}$
$z + w = 2 + 3i + 5 - 4i = 7 - i$
L.H.S. = $\overline{z+w} = 7 + i$
R.H.S. = $\bar{z} + \bar{w} = 2 - 3i + 5 + 4i = 7 + i$
Hence L.H.S. = R.H.S.

(iii) $\overline{zw} = \bar{z}\,\bar{w}$
$zw = (2 + 3i)(5 - 4i) = 10 - 8i + 15i - 12i^2 = 10 + 7i - 12(-1)$ $(\because i^2 = -1)$
$= 10 + 12 + 7i = 22 + 7i$
L.H.S. = $\overline{zw} = 22 - 7i$
R.H.S. = $\bar{z}\,\bar{w} = (2 - 3i)(5 + 4i) = 10 + 8i - 15i - 12i^2$
$= 10 - 7i - 12(-1) = 10 + 12 - 7i = 22 - 7i$
Hence L.H.S. = R.H.S.

(iv) $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$, where $w \neq 0$.
$$\frac{z}{w} = \frac{2+3i}{5-4i} = \frac{2+3i}{5-4i} \times \frac{5+4i}{5+4i} = \frac{(2+3i)(5+4i)}{25-16i^2} = \frac{10+8i+15i+12i^2}{25-16(-1)}$$
$$= \frac{10+23i-12}{25+16} = \frac{-2+23i}{41} = -\frac{2}{41} + \frac{23}{41}i \quad ; \quad (\because i^2 = -1)$$
L.H.S. = $\overline{\left(\frac{z}{w}\right)} = -\frac{2}{41} - \frac{23}{41}i$
R.H.S. = $\frac{\bar{z}}{\bar{w}} = \frac{2-3i}{5+4i} = \frac{2-3i}{5+4i} \times \frac{5-4i}{5-4i} = \frac{(2-3i)(5-4i)}{25-16i^2}$
$$= \frac{10-8i-15i+12i^2}{25-16(-1)} = \frac{10-12-23i}{25+16} = \frac{-2-23i}{41} = -\frac{2}{41} - \frac{23}{41}i$$
Hence L.H.S. = R.H.S.

(v) $\frac{1}{2}(z + \bar{z})$ is the real part of z.
$\frac{1}{2}(z + \bar{z}) = \frac{1}{2}(2 + 3i + 2 - 3i) = \frac{1}{2}(4) = 2$ is real part of z

Q1. Multiple Choice Questions. Choose the correct answer. ; Review EX #2 Q.1

(i) $(27x^{-1})^{-2/3}$ ………
(a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$ (c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $\frac{\sqrt{x^3}}{8}$

(ii) Write $\sqrt[7]{x}$ in exponential form……..
(a) x (b) x^7 (c) $x^{1/7}$ (d) $x^{7/2}$

(iii) Write $4^{2/3}$ with radical sign………
(a) $\sqrt[3]{4^2}$ (b) $\sqrt{4^3}$ (c) $\sqrt[3]{4^3}$ (d) $\sqrt{4^6}$

(iv) In $\sqrt[3]{35}$ the radicand is……..
(a) 3 (b) $\frac{1}{3}$ (c) 35 (d) none of these

(vi) The conjugate of $5+4i$ is........
(a) $-5+4i$ (b) $-5-4i$ (c) $5-4i$ (d) $5+4i$

(vii) The value of i^9 is........
(a) 1 (b) -1 (c) i (d) $-i$

(viii) Every real number is.........
(a) a positive integer (b) a rational number
(c) a negative integer (d) a complex number

(ix) Real part of $2ab(i+i^2)$ is ....
(a) $2ab$ (b) $-2ab$ (c) $2abi$ (d) $-2abi$

(x) Imaginary part of $-i(3i+2)$ is......
(a) -2 (b) 2 (c) 3 (d) -3

(xi) Which of the following sets have the closure property w.r.t. addition........,
(a) {0} (b) {0,-1} (c) {0,1} (d) $\{1, \sqrt{2}, 5\}$

(xii) Name the property of real numbers used in $\left(-\frac{\sqrt{5}}{2}\right)\times 1 = -\frac{\sqrt{5}}{2}$ ....
(a) additive identity (b) additive inverse
(c) multiplicative identity (d) multiplicative inverse

(xiii) If $z<0$ then $x<y \Rightarrow$......
(a) $xz<yz$ (b) $xz>yz$
(c) $xz=yz$ (d) none of these

(xiv) If $a,b \in R$ then only one of $a=b$ or $a<b$ or $a>b$ holds is called ......
(a) trichotomy property (b) transitive property
(c) additive property (d) multiplicative property

(xv) A non-terminating, non-recurring decimal represents:
(a) a natural number (b) a rational number
(c) an irrational number (d) a prime number

Answers:

i. a	ii. c	iii. a	iv. c	v. b	vi. c
vii. c	viii. d	ix. b	x. a	xi. a	xii. c
xiii. b	xiv. a	xv. c			

Q2. True or false? Identify. ; Review EX #2 Q.2
(i) Division is not an associative operation. ......
(ii) Every whole number is a natural number. ......
(iii) Multiplicative inverse of 0.02 is 50. ......
(iv) π is a rational number. ......
(v) Every integer is a rational number. ......
(vi) Subtraction is a commutative operation. ......
(vii) Every real number is a rational number. ......
(viii) Decimal representation of a rational number is either terminating or recurring. ......
(ix) $1.\overline{8} = 1+\frac{8}{9}$ ......

Answers:

(i) True (ii) False (iii) True (iv) False

GUESS PAPER & MODEL PAPER # 03
BASED ON UNIT # 3 (Reduced Syllabus)
LOGARITHMS

Unit 3	Logarithms
Exercise 3.1	Q1(i, ii, iv, vi); Q2(iii, iv)
Exercise 3.2	Q1(i, iii); Q3; Q4(ii, iv); Q5
Exercise 3.3	Q1(iv, v, vi); Q2; Q3(iii, iv); Q4
Exercise 3.4	Q1(i, iii, iv, vi); Q2; Q5
Review Ex 3	Q1; Q2

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) If $a^x = n$, then.........
(A) $a = log_x n$ (B) $x = log_n a$ (C) $x = log_a n$ (D) $a = log_n x$

(ii) The relation $y = \log_z x$ implies
(A) $x^y = z$ (B) $z^y = x$ (C) $x^z = y$ (D) $y^z = x$

(iii) The logarithm of unity to any base is........
(A) 1 (B) 10 (C) e (D) 0

(iv) The logarithm of any number to itself as base is.......
(A) 1 (B) 0 (C) -1 (D) 10

(v) log e =................, where $e \approx 2.718$
(A) 0 (B) 0.4343 (C) ∞ (D) 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ is........
(A) $\log p - \log q$ (B) $\frac{\log p}{\log q}$ (C) $\log p + \log q$ (D) $\log q - \log p$

(vii) $\log p - \log q$ is same as.......
(A) $\log\left(\frac{q}{p}\right)$ (B) $log(p-q)$ (C) $\frac{\log p}{\log q}$ (D) $\log\left(\frac{p}{q}\right)$

(ix) $\log_b a \times \log_c b$ can be written as.......
(A) $\log_a c$ (B) $\log_c a$ (C) $\log_a b$ (D) $\log_b c$

(x) $\log_y x$ will be equal to.......
(A) $\frac{\log_z x}{\log_y z}$ (B) $\frac{\log_x z}{\log_y z}$ (C) $\frac{\log_z x}{\log_z y}$ (D) $\frac{\log_z y}{\log_z x}$

(xi) For common logarithms, the base is......
(A) 1 (B) 10 (C) e (D) 0

(xii) The integral part of the common logarithm of a number is called the.......
(A) characteristics (B) Mantissa (C) Antilogarithm (D) Integer

(xiii) The decimal part of the common logarithm of a number is called the......
(A) characteristics (B) Mantissa (C) Antilogarithm (D) Integer

(xiv) If $x = \log y$, then y is called the..........of x.
(A) characteristics (B) Mantissa (C) Antilogarithm (D) Integer

(xv) If the characteristic of the logarithm of a number is 1, that number will have.........digits in its integral part.
(A) 1 (B) 10 (C) e (D) 2

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Express each of the following numbers in scientific notation.
(i) 5700 (ii) 49,800,000 ; EX #3.1 Q.1;(i , ii)

(ii) Express each of the following numbers in scientific notation.
(iv) 416.9 (vi) 0.00643 ; EX #3.1 Q.1;(iv , vi)

(iii) Express the following numbers in ordinary notation.
(iii) 9.018×10^{-6} (iv) 7.865×10^{8} ; EX #3.1 Q.2;(iii , iv)

(iv) Evaluate (i) $\log 2 \frac{1}{128}$ (ii) $\log 512$ to the base $2\sqrt{2}$; EX #3.2 Q.5

(v) Express $\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$ as a single logarithm. ; EX #3.3 Q.2

(vi) Calculate the following: (i) $\log_3 2 \times \log_2 81$ (ii) $\log_5 3 \times \log_3 25$; EX #3.3 Q.4

(vii) Use log tables to find the value of 0.8176×13.64 ; EX #3.4 Q.1;(i)

(viii) Use log tables to find the value of $\frac{0.678 \times 9.01}{0.0234}$; EX #3.4 Q.1;(iii)

(ix) Use log tables to find the value of $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$; EX #3.4 Q.1;(iv)

(x) Use log tables to find the value of $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$; EX #3.4 Q.1;(vi)

(xi) If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$. ; EX #3.4 Q.5

(xii) What replacement for the unknown in each of following will make the statement true?
(ii) $\log_a 6 = 0.5$ (iv) $10^p = 4$; EX #3.2 Q.4;(ii , iv)

(xiii) A gas is expanding according to the law $pV^n = C$. Find C when $p = 80$, $v = 3.1$ and $n = \frac{5}{4}$. ; EX #3.4 Q.2

(xiv) Write the following in the form of a single logarithm.
(iii) $2\log x - 3\log y$ (iv) $\log 5 + \log 6 - \log 2$; EX #3.3 Q.3;(iii , iv)

Q.3 Show whether or not the points coordinates $(1,3)$, $(4,2)$ and $(-2,6)$ are vertices of a right triangle. ; EX #9.2 ; Q.3

Q.4 One exterior angle formed on producing one side of a parallelogram is 40°. Find the measures of its interior angles. ; EX #11.1 ; Q.2

Q.5 Each point on the bisector of an angle is equidistant from its arms. ; Theorem # 12.1.4

Q.6 Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side? ; EX #13.1 ; Q.1

(a) 5 cm (b) 20 cm (c) 25 cm (d) 30 cm

Q.7 Construct a Δ equal in area to the quadrilateral ABCD, having $m\overline{AB} = 6cm$, $m\overline{BC} = 4cm$, $m\overline{AC} = 7.2\,cm$, $m\angle BAD = 105^o$ and $m\overline{BD} = 8cm$. ; EX #17.3 Q.3

SOLUTION OF GUESS PAPER & MODEL PAPER # 3
(Reduced Syllabus)

SECTION- A (MCQs)

i. C	ii. B	iii. D	iv. A	v. B	vi. A
vii. D	viii. C	ix. B	x. C	xi. B	xii. A
xiii. B	xiv. C	xv. D			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Express each of the following numbers in scientific notation.

(i) 5700 (ii) 49,800,000 ; EX #3.1 Q.1;(i , ii)

Solution: (i) 5700

$$= \frac{5700}{1000} \times 1000 = 5.7 \times 10^3$$

(ii) 498 00 000

$$= \frac{49800000}{10000000} \times 10000000 = 4.98 \times 10^7$$

Note: A number written in the form $a \times 10^n$, where $1 \le a \le 10$ and n is an integer, is called the scientific notation.

(ii) Express each of the following numbers in scientific notation.

(iv) 416.9 (vi) 0.00643 ; EX #3.1 Q.1;(iv , vi)

Solution: (iv) 416.9

$$\frac{4169}{10} = 4169 \times 10^{-1} \Rightarrow \frac{4169}{1000} \times 1000 \times 10^{-1} = 4.169 \times 10^{3-1} = 4.169 \times 10^2$$

(vi) 0.00643

$$\frac{00643}{100000} = 643 \times 10^{-5} \Rightarrow \frac{643}{100} \times 100 \times 10^{-5} = 6.43 \times 10^{2-5} = 6.43 \times 10^{-3}$$

(iii) Express the following numbers in ordinary notation.

(iii) 9.018×10^{-6} (iv) 7.865×10^{8} ; EX #3.1 Q.2;(iii , iv)

Solution: (iii) 9.018×10^{-6}

$$= \frac{9018}{1000} \times 10^{-6} = 9018 \times 10^{-6-3}$$

$$= 9018 \times 10^{-9} = \frac{9018}{1000000000} = 0.000009018$$

(iv) Evaluate (i) $\log 2 \frac{1}{128}$ (ii) log 512 to the base $2\sqrt{2}$; EX #3.2 Q.5

Solution: (i) $\log 2 \frac{1}{128}$; Let $\log 2 \frac{1}{128} = x$

Exponential form is $\therefore\ 2^x = \frac{1}{128} \Rightarrow \therefore\ 2^x = \frac{1}{2^7} \Rightarrow$ or $2^x = 2^{-7} \Rightarrow \quad x = -7$

(ii) log 512 to the base $2\sqrt{2}$

Let $\log_{2\sqrt{2}} 512 = x$; Exponential form is $(2\sqrt{2})^x = 512$

$\left(2 \times 2^{\frac{1}{2}}\right)^x = 2^9 \Rightarrow (2^{3/2})^x = 2^9 \Rightarrow \left(1 + \frac{1}{2} = \frac{3}{2}\right) \Rightarrow \quad 2^{3x/2} = 2^9 \Rightarrow \quad \frac{3x}{2} = 9$

$x = \frac{9 \times 2}{3} = \frac{18}{3} = 6$

(v) Express $\log x - 2\log x + 3\log(x+1) - \log(x^2 - 1)$ as a single logarithm. ; EX #3.3 Q.2

Solution: $\log x - 2\log x + 3\log(x+1) - \log(x^2 - 1$

$= (1-2)\log x + 3\log(x+1) - \log(x+1)(x-1)$

$= -\log x + 3\log(x+1) - [\log(x+1) + \log(x-1)]$

$= -\log x + 3\log(x+1) - \log(x+1) - \log(x-1) = \quad 2\log(x+1) - \log x - \log(x-1)$

$= 2\log(x+1) - \log[\log x + \log(x-1)] = \log(x+1)^2 - \log x(x-1)$

$= \log \frac{(x+1)^2}{x(x-1)}$

(vi) Calculate the following: (i) $\log_3 2 \times \log_2 81$ (ii) $\log_5 3 \times \log_3 25$; EX #3.3 Q.4

Solution: (i) $\log_3 2 \times \log_2 81$

$= \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2} = \frac{\log 81}{\log 3} = \frac{\log 3^4}{\log 3} = \frac{4\log 3}{\log 3} = 4$

(ii) $\log_5 3 \times \log_3 25$

$= \frac{\log 3}{\log 5} \times \frac{\log 25}{3} = \frac{\log 25}{\log 5} = \frac{\log 5^2}{\log 3} = \frac{2\log 5}{\log 5} = 2$

(vii) Use log tables to find the value of 0.8176×13.64 · EX #3.4 Q.1;(i)

Solution: 0.8176×13.64

Let $x = 0.8176 \times 13.64$

$\log x = \log(0.8176 \times 13.64) = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348 = -1 + 0.9125 + 1 + 0.1348$

$\log x = 0.9125 + 0.1348$

$\log x = 1.0473$

Taking antilog on both side

Antilog (log x) = Antilog (1.0473)

$x = 11.15$

(viii) Use log tables to find the value of $\frac{0.678 \times 9.01}{0.0234}$; EX #3.4 Q.1;(iii)

Solution: $\frac{0.678 \times 9.01}{0.0234}$

Let $x = \frac{0.678 \times 9.01}{0.0234}$

$\log x = \log \frac{0.678 \times 9.01}{0.0234} = \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - \bar{2}.3692 = -1 + 0.8312 + 0.9547 - \bar{2} - 0.3692$

$= -1 + 2 + 1.7859 - 0.3692 = 1 + 1.4167$

$\log x = 2.4167$

Taking antilog on both side

(ix) Use log tables to find the value of $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$; EX #3.4 Q.1;(iv)

Solution: $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

$x = (2.709)^{1/5} \times (1.239)^{1/7}$

$\log x = \log [(2.709)^{1/5} \times (1.239)^{1/7}]$

$= \log (2.709)^{1/5} + \log (1.239)^{1/7} = \frac{1}{5}(0.4328) + \frac{1}{7}\log 1.239$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931) = 0.08656 + 0.0133$

$= 0.09986$

$\log x = 0.0999$

Taking antilog on both side

Antilog (log x) = Antilog (0.999)

$x = 1.258$

(x) Use log tables to find the value of $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$; EX #3.4 Q.1;(vi)

Solution: $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Let $x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}} = \left(\frac{0.7214 \times 2.37}{60.8}\right)^{\frac{1}{3}}$

$\log x = \log \left(\frac{0.7214 \times 2.37}{60.8}\right)^{\frac{1}{3}}$

$= \frac{1}{3}\log\frac{0.7214 \times 20.37}{60.8} = \frac{1}{3}\{(\log 0.7214 \times \log 20.37) - \log 60.8\}$

$= \frac{1}{3}\{\log 0.7214 + \log 20.37 - \log 60.8\}$

$= \frac{1}{3}\{\bar{1}.8581 + 1.3090 - 1.7839\}$

$= \frac{1}{3}\{-1 + 0.8581 + 1 + 0.3090 - 1.7839\} = \frac{1}{3}\{1.1671 - 1.7839\}$

$= \frac{1}{3}\{-3 + 3 + 1.1671 - 1.7839\} = \frac{1}{3}\{-3 + 4.1671 - 1.7839\}$

$= \frac{1}{3}\{-3 + 2.3832\} = -1 + 0.7944$

$\log x = \bar{1}.7944$

Taking antilog on both side Antilog (log x) = Antilog ($\bar{1}.7944$)

$x = 0.6229$

(xi) If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$. ; EX #3.4 Q.5

Solution: $V = \frac{1}{3}\pi r^2 h$

By putting the values $V = \frac{1}{3} \times \frac{22}{7} \times 25^2 \times 4.2$

$V = 22 \times 25^2 \times 0.2$

$V = 22 \times 25^2 \times \frac{2}{10}$

$\log V = \log\left(22 \times 25^2 \times \frac{2}{10}\right)$

$= \quad 1.3424 + 2.7959 + 0.3010 - 1$

$\log V = 4.4393$

Taking antilog on both side $Antilog\,(\log V) = $ Antilog (4.4393)

So, $V = 27.50$

(xii) What replacement for the unknown in each of following will make the statement true?

(ii) $\log_a 6 = 0.5$ (iv) $10^p = 4$; EX #3.2 Q.4;(ii , iv)

Solution:

(ii) $\log_a 6 = 0.5 \Rightarrow a^{0.5} = 6 \Rightarrow a^{1/2} = 6 \Rightarrow \sqrt{a} = 6$

By squaring on both sides $a = 36$

(iv) $10^p = 4$

Taking log on both sides $\log 10^p = \log 4$

or $p \log 10 = \log 4 \Rightarrow p \times 1 = 0.6021$; $(\because \log 10 = 1)$

$p = 0.6021$

(xiii) A gas is expanding according to the law $pV^n = C$. Find C when $p = 80$, $v = 3.1$ and $n = \frac{5}{4}$. ; EX #3.4 Q.2

Solution: $pv^n = C$

Substituting $p = 80, \quad v = 3.1, \quad n = \frac{5}{4}$

$C = 80\,(3.1)^{\frac{5}{4}}$

$\log C = \log 80\,(3.1)^{\frac{5}{4}} = \log 80 + \frac{5}{4}\log 3.1$

$= 1.9031 + \frac{5}{4}(0.4914) = 1.9031 + \frac{2.570}{4} = 1.9031 + 0.6143$

$\log C = 2.5174$

Taking antilog on both side $Antilog\,(\log C) = $ Antilog (2.5174)

So, $C = 329.2$

(xiv) Write the following in the form of a single logarithm.

(iii) $2 \log x - 3 \log y$ (iv) $\log 5 + \log 6 - \log 2$; EX #3.3 Q.3;(iii , iv)

Solution: (iii) $2 \log x - 3 \log y = \log x^2 \times \log y^3 = \log \frac{x^2}{y^3}$

(iv) $\log 5 + \log 6 - \log 2 = \log 5 \times 6 - \log 2 = \log \frac{5 \times 6}{2}$

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Show whether or not the points coordinates $(1,3)$, $(4,2)$ and $(-2,6)$ are vertices of a right triangle. ; EX #9.2 ; Q.3

Solution: Let the given points be $A\,(1,3), B(4,2)$ and $C(-2,6)$.

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

$|BC| = \sqrt{(4+2)^2 + (2-6)^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$

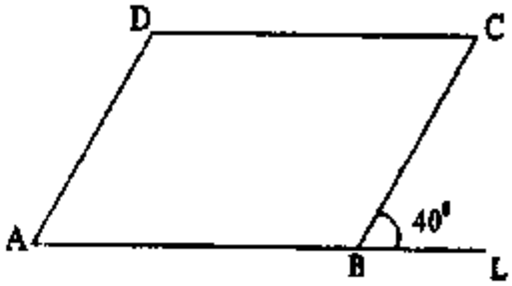
$|CA| = \sqrt{(1+2)^2 + (3-6)^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{9+9} = \sqrt{18}$

$\overline{|BC|^2} = 52$

$\overline{|AB|^2} + \overline{|CA|^2} = 10 + 18 = 28 \neq |BC|^2$

Since given points does not obey the Pythagoras theorem therefore the coordinates are not the vertices of right angle triangle.

$m\angle ABC + 40° = 180°$
∴ ABL is a straight line
∴ $m\angle ABC = 180° - 40° = 140°$
$m\angle D = m\angle ABC = 140°$
Opposite angles of a parallelogram
$m\angle D + m\angle C = 180°$
$140° + m\angle C = 180°$
∴ $m\angle C = 180° - 140° = 40°$
$m\angle A = m\angle C = 40°$
Opposite angles of parallelogram



So the measures of interior angles of the parallelogram are 140°, 40°, 140° and 40°.

Q.5 Each point on the bisector of an angle is equidistant from its arms. ; Theorem # 12.1.4

Solution:

Given:

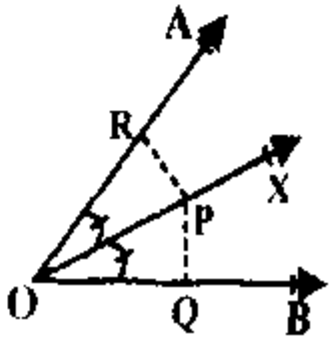
A point P is on $\overrightarrow{OX}$, the bisector of $\angle AOB$

To prove:

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction:

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$



Proof:

Statements	Reasons
In $\Delta POQ \leftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PRO \cong \angle PQO$	Construction
$\angle POQ \cong \angle POR$	Given
∴ $\Delta POQ \cong \Delta POR$	S.A.A. ≅ S.A.A.
and $\overline{PQ} \cong \overline{PR}$	Corresponding sides of congruent triangles

Q.6 Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side? ; EX #13.1 ; Q.1

(a) 5 cm (b) 20 cm (c) 25 cm (d) 30 cm

Solution: (a) Measure of sides are 10 cm, 15 cm and 5 cm

As $10 + 5 = 15$

Since the sum of two sides is equal to the third side therefore:

So 5 cm is not possible.

(b) Sides are 10 cm, 15 cm and 20 cm

$10 + 15 > 20$
$10 + 20 > 15$
$15 + 20 > 10$

Since the sum of two sides is greater than third side therefore:
20 cm is possible for third side.

(c) Sides are 10 cm, 15 cm and 25 cm

As $10 + 15 = 25$

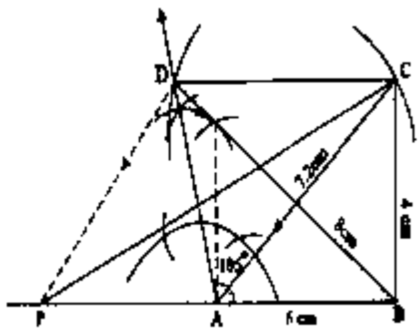
Since the sum of two sides is equal to the third side therefore:

So 25 cm is not possible.

(d) Sides are 10 cm, 15 cm and 30 cm

Q.7 Construct a Δ equal in area to the quadrilateral ABCD, having $m\overline{AB} = 6cm$, $m\overline{BC} = 4cm$, $m\overline{AC} = 7.2\ cm$, $m\angle BAD = 105^o$ and $m\overline{BD} = 8cm$. ; EX #17.3 Q.3

Solution:



Construction:

(i) Take $m\overline{AB} = 6cm$.
(ii) With centre at the end point A and radius 7.2 cm draw an arc.
(iii) With B as centre and radius 4 cm draw another arc to cut AL at the point D.
(iv) Join $\overline{AC}$ and $\overline{BC}$.
(v) At the end point A make $m\angle BAL = 105^o$
(vi) With B as centre and radius 8 cm draw arc to cut AL at the point D.
(vii) Join DC to complete the quadrilateral ABCD.
(viii) Draw $\overline{DP} \parallel \overline{CA}$ to meet BA produced at P.
(ix) Join P to C.
(x) Then PBC is the required triangle.

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Find the common logarithms of the following numbers.
(i) 232.92 (iii) 0.00032 ; EX #3.2 Q.1;(i , iii)

Solution: (i) 232.92

232.92 can be rounded off as 232.9. The characteristic is 2 as there are 3 digits.

To find mantissa we follow the row of 23 and reach the column of 2 to get 3655. In the same row in the difference column of 9 we see 17. Add 3655 and 17 and get mantissa .3672.

So log 232.92 = 2.3672

(iii) 0.000 32

The characteristic is -4 as which is written as $\bar{4}$.

To find mantissa we follow the row of 32 and reach the column of 0 to get 5051. So mantissa is 0.5051. So log 0.000 32 = $\bar{4}.5051$

Q3. Find the numbers whose common logarithms are (i) 3.5621 (ii) $\bar{1}.7427$; EX #3.2 Q.3

Solution: (i) 3.4521

Reading along the row corresponding to .56 we get 3648 at the intersection of this row and column of 2. The number at the intersection of this row and the mean difference column of 1 is 1. Adding 3648 and 1 we get 3649.

Since the characteristic is 3, the number has four digits. So the required number is 3649.

(ii) $\bar{1}.7427$

Reading along the row corresponding to .74 we get 5521 at the intersection of this row and column of 2. The number at the intersection of this row and the mean difference column of 7 is 9. Adding 5521 and 9 we get 5530.

Since the characteristics is $\bar{1}$. So the required number is 0.5530.

Q1. Write the following into sum or difference.

(iv) $\log \sqrt[3]{\frac{7}{\ }}$ (v) $\log \frac{(22)^{\frac{1}{3}}}{\ }$ (vi) $\log \frac{25 \times 47}{\ }$; EX #3.3 Q.1;(iv , v, vi)

$= \frac{1}{3}\log\frac{7}{15} = \frac{1}{3}[\log 7 - \log 15]$

(v) $\log\frac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} + \log 5^3 = \frac{1}{3}\log 22 - 3\log 5$

(vi) $\log\frac{25\times 47}{29} = \log(25\times 47) - \log 29 = \log 25 + \log 47 - \log 29$

Q1. Multiple Choice Questions. Choose the correct answer. ; **Review EX #3 Q.1**

(i) If $a^x = n$, then.........
(a) $a = \log_x n$ (b) $x = \log_n a$ (c) $x = \log_a n$ (d) $a = \log_n x$

(ii) The relation $y = \log_z x$ implies
(a) $x^y = z$ (b) $z^y = x$ (c) $x^z = y$ (d) $y^z = x$

(iii) The logarithm of unity to any base is........
(a) 1 (b) 10 (c) e (d) 0

(iv) The logarithm of any number to itself as base is.......
(a) 1 (b) 0 (c) -1 (d) 10

(v) log e =......, where $e \approx 2.718$
(a) 0 (b) 0.4343 (c) ∞ (d) 1

(vi) The value of $\log\left(\frac{p}{q}\right)$ is........
(a) $\log p - \log q$ (b) $\frac{\log p}{\log q}$ (c) $\log p + \log q$ (d) $\log q - \log p$

(vii) $\log p - \log q$ is same as........
(a) $\log\left(\frac{q}{p}\right)$ (b) $\log(p-q)$ (c) $\frac{\log p}{\log q}$ (d) $\log\left(\frac{p}{q}\right)$

(viii) $\log(m^n)$ can be written as........
(a) $(\log m)^n$ (b) $m\log n$ (c) $n\log m$ (d) $\log(mn)$

(ix) $\log_b a \times \log_c b$ can be written as.......
(a) $\log_a c$ (b) $\log_c a$ (c) $\log_a b$ (d) $\log_b c$

(x) $\log_y x$ will be equal to........
(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$ (c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$

Solution:

(i) c	(ii) b	(iii) d	(iv) a	(v) b
(vi) a	(vii) d	(viii) c	(ix) b	(x) c

Q2. Complete the following. ; Review EX #3 Q.2

(i) For common logarithms, the base is......
(ii) The integral part of the common logarithm of a number is called the.......
(iii) The decimal part of the common logarithm of a number is called the......
(iv) If $x = \log y$, then y is called the...........of x.
(v) f the characteristic of the logarithm of a number is $\overline{2}$, that number will have.............zero(s) immediately after the decimal point.
(vi) If the characteristic of the logarithm of a number is 1, that number will have..........digits in its integral part.

Answers:

(i) 10 (ii) Characteristic (iii) Mantissa

GUESS PAPER & MODEL PAPER # 04
BASED ON UNIT # 4 (Reduced Syllabus)
ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS

Unit 4	Algebraic Expressions and Algebraic Formulas
Exercise 4.1	Q1; Q2; Q3(iii, iv, v, vii, viii); Q5(ii, iv, vi); Q6(ii, iii, iv, v)
Exercise 4.2	Q1; Q2; Q4; Q6; Q8; Q10; Q13; Q15(i, ii, iii)
Exercise 4.3	Q1(iii, iv); Q2(ii, iii); Q3(i, ii); Q4(iii, v)
Exercise 4.4	Q1(iii, iv, vii); Q2(i, ii); Q3(i); Q4(i, ii); Q5(ii)
Review Ex 4	Q1; Q2

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) $4x + 3y - 2$ is an algebraic........
(A) expression (B) sentence (C) equation (D) in equation

(ii) The degree of polynomial $4x^4 + 2x^2y$ is........
(A) 1 (B) 2 (C) 3 (D) 4

(iii) $a^3 + b^3$ is equal to........
(A) $(a-b)(a^2+ab+b^2)$ (B) $(a+b)(a^2-ab+b^2)$
(C) $(a-b)(a^2-ab+b^2)$ (D) $(a-b)(a^2+ab-b^2)$

(iv) $(3+\sqrt{2})(3+\sqrt{2})$ is equal to........
(A) 7 (B) -7 (C) -1 (D) 1

(v) Conjugate of surd $a+\sqrt{b}$ is........
(A) $-a+\sqrt{b}$ (B) $a-\sqrt{b}$ (C) $\sqrt{a}+\sqrt{b}$ (D) $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{\ } - \frac{1}{\ }$ is equal to ....

(vii) $\frac{a^2-b^2}{a+b}$ is equal to......
(A) $(a-b)^2$ (B) $(a+b)^2$ (C) $a+b$ (D) $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ is equal to......
(A) a^2+b^2 (B) a^2-b^2 (C) $a-b$ (D) $a+b$

(ix) The degree of the polynomial $x^2y^2+3xy+y^3$ is.........
(A) 1 (B) 2 (C) 3 (D) 4

(x) x^2-4 =.........
(A) $(x-2)(x-2)$ (B) $(x+2)(x-2)$
(C) $(x+2)(x+2)$ (D) $x+2$

(xi) $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)$(.........).
(A) $x^2-1+\frac{1}{x^2}$ (B) $x^2+1+\frac{1}{x^2}$ (C) $x^2-2+\frac{1}{x^2}$ (D) $x^2-1-\frac{1}{x^2}$

(xii) $2\left(a^2+b^2\right)=(a+b)^2+(.........)^2$.
(A) a^2+b^2 (B) $(a+b)^2(a+b)^2$
(C) $(a+b)^2(a-b)^2$ (D) $a+b$

(xiii) $\left(x-\frac{1}{x}\right)^2$ = ...
(A) $x^2-1+\frac{1}{x^2}$ (B) $x^2+1+\frac{1}{x^2}$ (C) $x^2-2+\frac{1}{x^2}$ (D) $x^2-1-\frac{1}{x^2}$

(xiv) Order of surd $\sqrt[3]{x}$ is ....
(A) 1 (B) 2 (C) 3 (D) 4

(xv) $\frac{1}{2-\sqrt{3}}$ = ...
(A) $2+\sqrt{3}$ (B) $2-\sqrt{3}$ (C) $-2-\sqrt{3}$ (D) $2-\sqrt{-3}$

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9\times 4=36)$

(i) Rationalize the denominator of the following. $\frac{6}{\sqrt{8}\sqrt{27}}$; EX #4.4 Q.1;(iii)

(ii) Simplify $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$; EX #4.4 Q.4;(i)

(iii) Simplify $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$; EX #4.4 Q.4;(ii)

(iv) If $\left(5x-\frac{1}{5x}\right)=6$, then find the value of $\left(125x^3-\frac{1}{125x^3}\right)$; EX #4.2 Q.13

(v) If $x+\frac{1}{x}=3$, then find the value of $x^3+\frac{1}{x^3}$. ; EX #4.2 Q.10

(vi) If $x-y=4$ and $xy=21$, then find the value of x^3-y^3. ; EX #4.2 Q.8

(vii) If $x+y=7$ and $xy=12$, then find the value of $x^3+y^3+z^3$. ; EX #4.2 Q.6

(viii) If $x+y+z=78$ and $xy+yz+zx=59$, find the value of $x+y+z$. ; EX #4.2 Q.4

(ix) If $a+b=5$, $a-b=\sqrt{17}$, then find the value of ab. ; EX #4.2 Q.1

(x) If $a^2+b^2+c^2=45$ and $a+b+c=-1$, find the value of $ab+be+ca$. ; EX #4.2 Q.2

(xiii) Perform the indicated operation and simplify.

$$\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$$; EX #4.1 Q.5;(vi)

(xiv) If $a + b = 10$ and $a - b = 6$, then find the value of $(a^2 + b^2)$. ; EX #4.2 Q.1

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Use the distance formula to prove whether or not the points $(1, 1)$, $(-2, -8)$ and $(4, 10)$ lie on a straight line? ; EX #9.2 ; Q.4

Q.4 If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it. ; Theorem # 13.1.1

Q.5 A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach? ; EX #15; Q.8

Q.6 Construct a right-angled Δ measure of whose hypotenuse is 5 cm and one side is 3.2 cm. ; EX #17.1 Q.3

Q.7 Construct a Δ with sides 4 cm, 5 cm and 6 cm and construct a rectangle having its area equal to that of the Δ. Measure its diagonals. Are they equal? ; EX #17.4 Q.1

SOLUTION OF GUESS PAPER & MODEL PAPER # 4
(Reduced Syllabus)

SECTION- A (MCQs)

i. A	ii. D	iii. B	iv. A	v. B	vi. B
vii. B	viii. C	ix. D	x. B	xi. A	xii. C
xiii. C	xiv. C	xv. A			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Rationalize the denominator of the following. $\frac{6}{\sqrt{8}\sqrt{27}}$; EX #4.4 Q.1;(iii)

Solution: $\frac{6}{\sqrt{8}\sqrt{27}}$

$$= \frac{6}{\sqrt{8}\sqrt{27}} \times \frac{\sqrt{8}\sqrt{27}}{\sqrt{8}\sqrt{27}} = \frac{6}{8\times 27}.\sqrt{8}\sqrt{27} = \frac{1}{36}.\sqrt{4.2}.\sqrt{9.3}$$

$$= \frac{1}{4\times 9} \times 2 \times 3 . \sqrt{2}\sqrt{3} = \frac{1}{6}\sqrt{6} = \frac{\sqrt{6}}{6}$$

(ii) Simplify $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$; EX #4.4 Q.4;(i)

Solution: $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{2\sqrt{5}-2\sqrt{6}}{5-3} = \frac{2(\sqrt{5}-\sqrt{6})}{2} = \sqrt{5}-\sqrt{6}$$

$= \frac{2-\sqrt{3}}{4-3} + \frac{2(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2-\sqrt{5}}{4-5} = \frac{2-\sqrt{3}}{1} + \frac{2(\sqrt{5}+\sqrt{3})}{2} + \frac{2-\sqrt{5}}{-1}$

$= 2 - \sqrt{3} + \sqrt{5} + \sqrt{3} - 2 + \sqrt{5} \quad = 2\sqrt{5}$

(iv) If $\left(5x - \frac{1}{5x}\right) = 6$, then find the value of $\left(125x^3 - \frac{1}{125x^3}\right)$; EX #4.2 Q.13

Solution: $\left(5x - \frac{1}{5x}\right) = 6$

$\left(5x - \frac{1}{5x}\right)^3 = (5x)^3 - \left(\frac{1}{5x}\right)^3 - 3(5x)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right)$

$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6) \Rightarrow 216 = 125x^3 - \frac{1}{125x^3} - 18$

So $125x^3 - \frac{1}{125x^3} = 216 + 18 \Rightarrow 125x^3 - \frac{1}{125x^3} = 234$

(v) If $x + \frac{1}{x} = 3$, then find the value of $x^3 + \frac{1}{x^3}$. ; EX #4.2 Q.10

Solution: $x + \frac{1}{x} = 3$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) \Rightarrow (3)^3 = x^3 + \frac{1}{x^3} + 3(3)$

$27 = x^3 + \frac{1}{x^3} + 9 \Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 = 18$

(vi) If $x - y = 4$ and $xy = 21$, then find the value of $x^3 - y^3$. ; EX #4.2 Q.8

Solution: $x - y = 4, \quad xy = 21$

$(x-y)^3 = x^3 - y^3 - 3xy(x-y) \Rightarrow (4)^3 = x^3 - y^3 - 3(21)(4)$

$64 = x^3 - y^3 - 252 \Rightarrow x^3 - y^3 = 64 + 252 = 316$

(vii) If $x + y = 7$ and $xy = 12$, then find the value of $x^3 + y^3 + z^3$. ; EX #4.2 Q.6

Solution: $x + y = 7, \quad xy = 12$

$(x+y)^3 = x^3 + y^3 + 3xy(x+y) \Rightarrow (7)^3 = x^3 + y^3 + 3(12)(7)$

$343 = x^3 + y^3 + 252 \Rightarrow x^3 + y^3 = 343 - 252 = 91$

(viii) If $x + y + z = 78$ and $xy + yz + zx = 59$, find the value of $x + y + z$. ; EX #4.2 Q.4

Solution: $x^2 + y^2 + z^2 = 78$, $xy + yz + zx = 59$

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy+yz+zx) = 78 + 2(59)$

$(x+y+z)^2 = 78 + 118 = 196$

$\Rightarrow \quad x + y + z = \pm\sqrt{196} = \pm 14$

(ix) If $a + b = 5$, $a - b = \sqrt{17}$, then find the value of ab. ; EX #4.2 Q.1

Solution: $a + b = 5, \quad a - b = \sqrt{17}$

$(a+b)^2 - (a-b)^2 = 4ab \Rightarrow (5)^2 - (\sqrt{17})^2 = 4ab$

$25 - 17 = 4ab \Rightarrow 4ab = 8 \Rightarrow ab = 2$

(x) If $a^2 + b^2 + c^2 = 45$ and $a + b + c = -1$, find the value of $ab + be + ca$. ; EX #4.2 Q.2

Solution: $a^2 + b^2 + c^2 = 45, \quad a + b + c = -1$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

$(-1)^2 = 45 + 2(ab+bc+ca) \Rightarrow 1 = 45 + 2(ab+bc+ca)$

$1 - 45 = 2(ab+bc+ca) \Rightarrow 2(ab+bc+ca) = -44 \Rightarrow ab + bc + ca = -22$

$$= \frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)} = \frac{1+4x+4x^2-(1-4x+4x^2)}{(1-2x)(1+2x)} = \frac{1+4x+4x^2-1+4x-4x^2}{(1-2x)(1+2x)} = \frac{8x}{1-4x^2}$$

(xii) Perform the indicated operation and simplify. $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$; EX #4.1 Q.5 (iv)

Solution: $\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$

$$= \frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{(x+y)(x-y)} = \frac{x(x+y)-y(x-y)-2xy}{(x+y)(x-y)}$$

$$= \frac{x^2+xy-xy+y^2-2xy}{(x+y)(x-y)} = \frac{x^2+y^2-2xy}{(x+y)(x-y)} = \frac{(x-y)^2}{(x+y)(x-y)} = \frac{x-y}{x+y}$$

(xiii) Perform the indicated operation and simplify.

$\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1}$; EX #4.1 Q.5;(vi)

Solution: $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{x^4-1} = \frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}-\frac{4}{(x^2+1)(x+1)(x-1)}$

$$= \frac{(x^2+1)(x+1)-(x^2+1)(x-1)-2(x+1)(x-1)-4}{(x^2+1)(x+1)(x-1)} = \frac{x^3+x^2+x+1-(x^3-x^2+x-1)-2(x^2-1)-4}{x^4-1}$$

$$= \frac{x^3+x^2+x+1-x^3+x^2-x+1-2x^2+2-4}{x^4-1} = \frac{0}{x^4-1} = 0$$

(xiv) If $a+b=10$ and $a-b=6$, then find the value of (a^2+b^2). ; EX #4.2 Q.1

Solution: $a+b=10,\quad a-b=6$

$(a+b)^2+(a-b)^2=2(a^2+b^2) \Rightarrow (10)^2+(6)^2=2(a^2+b^2)$

$100+36=2(a^2+b^2) \Rightarrow 2(a^2+b^2)=136 \Rightarrow a^2+b^2=68$

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3\times 8=24)$

Q.3 Use the distance formula to prove whether or not the points $(1,1)$, $(-2,-8)$ and $(4,10)$ lie on a straight line? ; EX #9.2 ; Q.4

Solution: Let the given points be $A(1,1)$, $B(-2,8)$ and $(4,10)$.

Distance formula $= d = \pm\sqrt{|x_2-x_1|^2+|y_2-y_1|^2}$

$|AB| = \sqrt{(1+2)^2+(1+8)^2} = \sqrt{(3)^2+(9)^2} = \sqrt{9+81} = \sqrt{90} = \sqrt{9\times 10} = 3\sqrt{10}$

$|BC| = \sqrt{(4+2)^2+(10+8)^2} = \sqrt{(6)^2+(18)^2} = \sqrt{36+324} = \sqrt{360} = 6\sqrt{10}$

$|AC| = \sqrt{(4-1)^2+(10-1)^2} = \sqrt{(3)^2+(9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$

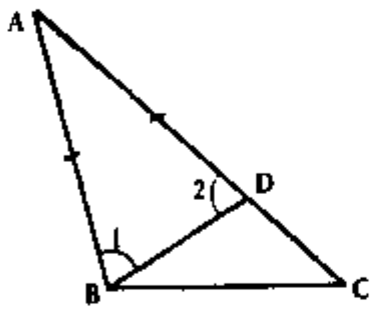
By applying the condition of collinear points

As $|AB|+|AC| = 3\sqrt{10}+3\sqrt{10} = 6\sqrt{10} = |BC|$

So the points A, B, C are on the same straight line. OR the given points are collinear

Q.4 If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it. ; Theorem # 13.1.1

Solution:



Given:

In ΔABC,

$m\overline{AC} > m\overline{AB}$

To prove:

$m\angle ABC > m\angle ACB$

Proof:

Statements	Reasons
In ΔABD $m\angle 1 = m\angle 2$ .... (i)	Angles opposite to congruent sides;
In ΔBCD $m\angle 2 > m\angle ACB$ .... (ii)	An exterior angle of triangle is greater than every non adjacent interior angle.
$\therefore$ $m\angle 1 > m\angle ACB$ ...... (iii) $m\angle ABC = m\angle 1 + m\angle DBC$	By (i) and (ii) Postulate of addition of measure of angles.
$\therefore$ $m\angle ABC > m\angle 1$ .... (iv) or $m\angle ABC > m\angle ACB$	By (iii) and (iv) Transitive property of inequality of real numbers.

Q.5 **A ladder 17 m long rests against a vertical wall. The foot of the ladder is 8 m away from the base of the wall. How high up the wall will the ladder reach? ; EX #15; Q.8**

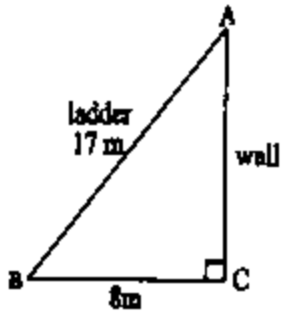
Solution: By Pythagoras Theorem

$$(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$$
$$(17)^2 = (m\overline{AC})^2 + (8)^2$$

$\therefore$ $(m\overline{AC})^2 = (17)^2 - (8)^2$

$= 289 - 64 = 225$

$m\overline{AC} = \sqrt{225} = 15\,cm$

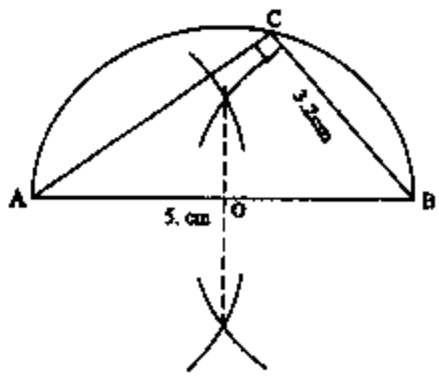


Q.6 **Construct a right-angled Δ measure of whose hypotenuse is 5 cm and one side is 3.2 cm. (Hint: Angle in a semi-circle is a right angle). ; EX #17.1 Q.3**

Solution:

Construction:

(i) Draw a line segment $m\overline{AB} = 5.2$ cm.
(ii) Find the mid-point O of $\overline{AB}$.
(iii) With centre at O and radius equal to $\overline{OA}$ draw and semi circle.



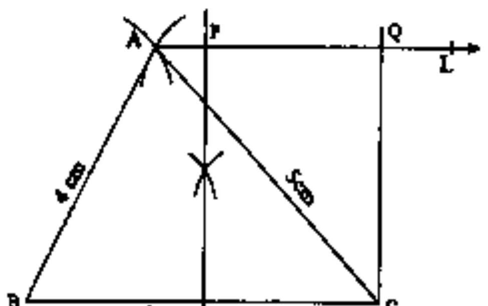
(iv) Join C to A and B.
Then ABC is the required triangle.

Q.7 **Construct a Δ with sides 4 cm, 5 cm and 6 cm and construct a rectangle having its area equal to that of the Δ. Measure its diagonals. Are they equal? ; EX #17.4 Q.1**

Solution:

Construction:

(i) Draw a line segment $m\overline{BC} = 6\,cm$.
(ii) With centre at the point B and radius as 4 cm draw an arc.
(iii) With centre at the point C with radius 5 cm draw w another arc cut the first arc at the point A.



(vii) Draw perpendicular $\overline{DP}$ to meet AL at P.
(viii) Cut off $\overline{PQ} = \overline{DC}$
(ix) Join Q to C.
Then PQCD is the required rectangle. Measure the diagonal $\overline{DQ} = 4.5\ cm$

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Identify whether the following algebraic expressions are polynomials (yes or not).

(i) $3x^2 + \frac{1}{x} - 5$ (ii) $3x^3 - 4x^2 - x\sqrt{x} + 3$
(iii) $x^2 - 3x + \sqrt{2}$ (iv) $\frac{3x}{2x-1} + 8$; EX #4.1 Q.1

Solution:

(i) No	(ii) No	(iii) Yes	(iv) No

Q2. State whether each of the following expression is a rational expression or not.

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$ (ii) $\frac{x^3-2x^2+\sqrt{3}}{2+3x-x^2}$ (iii) $\frac{x^2+6x+9}{x^2-9}$ (iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$; EX #4.1 Q.2

Solution:

(i) No	(ii) Yes	(iii) Yes	(iv) No

Q3. Reduce the following rational expressions to the lowest forms.

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$ (iv) $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$
(v) $\frac{(x+1)(x^2-1)}{(x+1)(x^2-4)}$ (vii) $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$ (viii) $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

EX #4.1 Q.3;(iii , iv , v, vii , viii)

Solution: (iii) $\frac{(x+y)^2-4xy}{(x-y)^2} = \frac{x^2+y^2+2xy-4xy}{(x-y)^2}$

$= \frac{x^2+y^2-2xy}{(x-y)^2} = \frac{(x-y)^2}{(x-y)^2} = 1$

(iv) $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)} = \frac{(x^3-y^3)(x-y)^2}{(x-y)(x^2+xy+y^2)} = \frac{(x-y)(x^2+xy+y^2)(x-y)^2}{(x-y)(x^2+xy+y^2)} = (x-y)^2$

(v) $\frac{(x+1)(x^2-1)}{(x+1)(x^2-4)} = \frac{(x+2)(x+1)(x-1)}{(x+1)(x+2)(x-1)} = \frac{x-1}{x-2}$

(vii) $\frac{64x^5-64x}{(8x^2+8)(2x+2)} = \frac{64x(x^4-1)}{8(x^2+1)2(x+1)} = \frac{64x(x^2+1)(x^2-1)}{16(x^2+1)(x+1)}$

$= \frac{4x(x^2-1)}{x+1} = \frac{4x(x+1)(x-1)}{x+1} = 4x(x-1)$

(viii) $\frac{9x^2-(x^2-4)^2}{4+3x-x^2} = \frac{(3x)^2-(x^2-4)^2}{4+3x-x^2} = \frac{[3x+(x^2-4)][3x-(x^2-4)]}{4+3x-x^2}$

$= \frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2} = \frac{(x^2+3x-4)(4+3x-x^2)}{4+3x-x^2} = x^2+3x-4$

(iv) $\frac{x^2-1}{x^2+2x+1}\cdot\frac{x+5}{1-x}$ (v) $\frac{x^2+xy}{y(x+y)}\cdot\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$; EX #4.1 Q.6;(ii , iii , iv, v)

Solution: (ii) $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}=\frac{4x-12}{x^2-9}\times\frac{x^2+6x+9}{18-2x^2}=\frac{4(x-3)}{(x+3)(x-3)}\times\frac{(x+3)^3}{2(9-x^2)}$

$=\frac{2(x-3)\times(x+3)\times(x+3)}{(x+3)(x-3)(3-x)(3+x)}=\frac{-2(3-x)(3+x)(x+3)}{(x+3)(x-3)(3-x)(x+3)}=\frac{-2}{x-3}=\frac{2}{3-x}$

(iii) $\frac{x^6-y^6}{x^2-y^2}\div(x^4+x^2y^2+y^4)$

$=\frac{x^6-y^6}{x^2-y^2}\times\frac{1}{x^4+x^2y^2+y^4}=\frac{(x^3+y^3)(x^3-y^3)}{(x+y)(x-y)}\times\frac{1}{x^4+2x^2y^2+y^4-x^2y^2}$

$=\frac{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)[(x^2+y^2)^2-(xy)^2]}=\frac{(x^2-xy+y^2)(x^2+xy+y^2)}{(x^2+xy+y^2)(x^2-xy+y^2)}=1$

(iv) $\frac{x^2-1}{x^2+2x+1}\cdot\frac{x+5}{1-x}=\frac{(x+1)(x-1)}{(x+1)^2}\cdot\frac{x+5}{1-x}=\frac{-(x+1)(1-x)(x+5)}{(x+1)(x+1)(1-x)}=\frac{-(x+5)}{x+1}$

(v) $\frac{x^2+xy}{y(x+y)}\cdot\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}=\frac{x^2+xy}{y(x+y)}\cdot\frac{x^2+xy}{y(x+y)}\cdot\frac{xy-2y}{x^2-x}=\frac{x(x+y)}{y(x+y)}\cdot\frac{x(x+y)}{y(x+y)}\cdot\frac{y(x-2)}{x(x-1)}=\frac{x(x-2)}{y(x-1)}$

Q15. Find the products, using formulas.

(i) $(x^2+y^2)(x^4-x^2y^2+y^4)$. (ii) $(x^3-y^3)(x^6+x^3y^3+y^6)$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

; EX #4.2 Q.15;(i , ii, iii)

Solution: (i) $(x^2+y^2)(x^4-x^2y^2+y^4)$

$=[x^2+y^2]\left[(x^2)^2-x^2.y^2+(y^2)^2\right]=(x^2)^3+(y^2)^3=x^6+y^6$

(ii) $(x^3-y^3)(x^6+x^3y^3+y^6)=[x^3-y^3]\left[(x^3)^2+x^3.y^3+(y^3)^2\right]=(x^3)^3-(y^3)^3=x^9-y^9$

(iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

$=[(x-y)(x^2+xy+y^2)][(x+y)(x^2-xy+y^2)][(x^2+y^2)(x^4-x^2y^2+y^4)]$

$=(x^3-y^3)(x^3+y^3)[(x^2)^3+(y^2)^3]=[(x^3)^2-(y^3)^2](x^6+y^6)$

$=[x^6-y^6](x^6+y^6)=(x^6)^2-(y^6)^2=x^{12}-y^{12}$

Q1. Express each of the following surds in the simplest form.

(iii) $\frac{3}{4}\sqrt[3]{128}$ (iv) $\sqrt[5]{96x^6y^7z^8}$; EX #4.3 Q.1;(iii, iv)

Solution: (iii) $\frac{3}{4}\sqrt[3]{128}$

$=\frac{3}{4}\sqrt[3]{64\times2}=\frac{3}{4}\sqrt[3]{4^3\times2}=\frac{3}{4}.4.\sqrt[3]{2}=3.\sqrt[3]{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

$=\sqrt[5]{32.3.x^5.x.y^5.y^3.z^5.z^3}=\sqrt[5]{(2xyz)^5.3xy^2z^3}=2xyz\sqrt[5]{3xy^2z^3}$

Q2. Simplify (ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$ (iii) $\sqrt[5]{243x^5y^{10}z^{15}}$; EX #4.3 Q.2;(ii, iii)

Solution: (ii) $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}=\frac{\sqrt{21\times9}}{\sqrt{63}}=\frac{\sqrt{189}}{\sqrt{63}}=\sqrt{\frac{189}{63}}=\sqrt{3}$

(iii) $\sqrt[5]{243x^5y^{10}z^{15}}=\left(3^5x^5y^{10}z^{15}\right)^{\frac{1}{5}}=3xy^2z^3$

Q3. Simplify by combining similar terms.

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} \quad = (3 - 6 + 4)\sqrt{5} = \sqrt{5}$

(ii) $4\sqrt{12} + 5\sqrt{27} - 3\sqrt{75} + \sqrt{300} \quad = 4\sqrt{4 \times 3} + 5\sqrt{9 \times 3} - 3\sqrt{25 \times 3} + \sqrt{100 \times 3}$

$= 4 \times 2 \times \sqrt{3} + 5 \times 3 \times \sqrt{3} - 3 \times 5 \times \sqrt{3} + 10 \times \sqrt{3}$

$= 8\sqrt{3} + 15\sqrt{3} - 15\sqrt{3} + 10\sqrt{3} \quad = (8 + 15 - 15 + 10)\sqrt{3} \quad = 18\sqrt{3}$

Q4. Simplify

(iii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ **(v)** $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2)$ **; EX #4.3 Q.4;(iii, v)**

Solution: (iii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) \quad = (\sqrt{5})^2 - (\sqrt{3})^2 \quad = 5 - 3 = 2$

(v) $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})(x + y)(x^2 + y^2) = \left[(\sqrt{x})^2 - (\sqrt{y})^2\right](x + y)(x^2 + y^2)$

$= (x - y)(x + y)(x^2 + y^2) \quad = (x^2 - y^2)(x^2 + y^2) \quad = (x^2)^2 - (y^2)^2 \quad = x^4 - y^4$

Q1. Rationalize the denominator of the following. (iv) $\frac{1}{3+2\sqrt{5}}$ **; EX #4.4 Q.1;(iv)**

Solution: $= \frac{1}{3+2\sqrt{5}} \cdot \frac{3-2\sqrt{5}}{3-2\sqrt{5}} \quad = \frac{3-2\sqrt{5}}{(3)^2 - (2\sqrt{5})^2} = \frac{3-2\sqrt{5}}{9-20} \quad = \frac{3-2\sqrt{5}}{-11} \quad = -\frac{1}{11}\left(3 - 2\sqrt{5}\right)$

Q1. Rationalize the denominator of the following. (vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ **; EX #4.4 Q.1;(vii)**

Solution: $= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \quad = \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - (1)^2} = \frac{(\sqrt{3})^2 - 2\sqrt{3} + (1)^2}{3-1}$

$= \frac{3-2\sqrt{2}+1}{2} = \frac{4-2\sqrt{3}}{2} \quad = \frac{2(2-\sqrt{3})}{2} = 2 - \sqrt{3}$

Q2. Find the conjugate of $x + \sqrt{y}$. **(i)** $3 + \sqrt{7}$ **(ii)** $4 - \sqrt{5}$ **; EX #4.4 Q.2;(i, ii)**

Solution: (i) Conjugate of $3 + \sqrt{7}$ **is** $3 - \sqrt{7}$. **(ii) Conjugate of** $4 - \sqrt{5}$ **is** $4 + \sqrt{5}$.

Q3. (i) If $x = 2 - \sqrt{3}$, **find** $\frac{1}{x}$ **; EX #4.4 Q.3;(i)**

Solution: (i) $x = 2 - \sqrt{3}$

$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \quad = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \quad = \frac{2+\sqrt{3}}{(2)^2 - (\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} \quad = \frac{2+\sqrt{3}}{1} = 2 + \sqrt{3}$

Q5. (ii) Q14. If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$, **find the value of** $x + \frac{1}{x}$, $x^2 + \frac{1}{x^2}$ **and** $x^3 + \frac{1}{x^3}$.
; EX #4.4 Q.5;(ii)

Solution: $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \Rightarrow \quad x + \frac{1}{x} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} + \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \quad = \frac{(\sqrt{5}-\sqrt{2})^2 + (\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2 + (2)^2}$

$= \frac{(\sqrt{5})^2 + (\sqrt{2})^2 + (\sqrt{5})^2 + (\sqrt{2})^2}{5-2} \quad = \frac{5+2+5+2}{3} = \frac{14}{3}$

$\left(x + \frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2 \Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = \frac{196}{9}$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = \frac{196}{9} - 2 \quad = \frac{196-18}{9} = \frac{178}{9}$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = \left(\frac{14}{3}\right)^3 - 3\left(\frac{14}{3}\right)$

$= \frac{2744}{27} - \frac{14}{1} \quad = \frac{2744-378}{27} = \frac{2366}{27}$

Q1. Multiple Choice Questions. Choose the correct answers. ; Review EX #4 Q.1

(i) $4x + 3y - 2$ is an algebraic

(iii) $a^3 + b^3$ is equal to.........
(a) $(a-b)(a^2+ab+b^2)$ (b) $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$ (d) $(a-b)(a^2+ab-b^2)$

(iv) $(3+\sqrt{2})(3+\sqrt{2})$ is equal to.........
(a) 7 (b) -7 (c) -1 (d) 1

(v) Conjugate of surd $a+\sqrt{b}$ is.........
(a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$ (c) $\sqrt{a}+\sqrt{b}$ (d) $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{a-b} - \frac{1}{a+b}$ is equal to ....
(a) $\frac{2a}{a^2-b^2}$ (b) $\frac{2b}{a^2-b^2}$ (c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ is equal to......
(a) $(a-b)^2$ (b) $(a+b)^2$ (c) $a+b$ (d) $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ is equal to......
(a) a^2+b^2 (b) a^2-b^2 (c) $a-b$ (d) $a+b$

Answers:

(i) a	(ii) d	(iii) b	(iv) a
(v) b	(vi) b	(vii) b	(viii) c

Q2. Fill in the blanks. ; Review EX #4 Q.2

(i) The degree of the polynomial $x^2y^2 + 3xy + y^3$ is.........

(ii) $x^2 - 4 =$.........

(iii) $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)(.........)$.

(iv) $2\left(a^2+b^2\right) = (a+b)^2 + (.........)^2$

(v) $\left(x - \frac{1}{x}\right)^2 =$ ....

(vi) *Order of surd* $\sqrt[3]{x}$ *is* ....

(vii) $\frac{1}{2-\sqrt{3}} =$ ....

Answers:
(i) 4 (ii) $(x+2)(x-2)$ (iii) $x^2 - 1 + \frac{1}{x^2}$
(iv) $(a+b)^2(a-b)^2$ (v) $x^2 - 2 + \frac{1}{x^2}$ (vi) 3
(vii) $2+\sqrt{3}$

GUESS PAPER & MODEL PAPER # 05
BASED ON UNIT # 5 (Reduced Syllabus)
FACTORIZATION

Unit 5	Factorization
Exercise 5.1	Q1(i, v, vi); Q2(iii, iv); Q3(i, iii); Q4(ii, iv); Q5(i, ii, iii)
Exercise 5.2	Q1(i, iv, v); Q2(i, iii); Q3(ii, v, viii); Q4(i, iii, v); Q5(i, ii); Q6(i, ii)
Exercise 5.3	Q1(i, iii); Q2(i); Q3(i); Q5; Q7; Q9
Review Ex 5	Q1; Q2

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) The factors of $x^2 - 5x + 6$ are.....
(A) $x+1, x-6$ (B) $x-2, x-3$
(C) $x+6, x-1$ (D) $x+2, x+3$

(ii) Factors of $8x^3 + 27y^3$ are.......
(A) $(2x+3y), (4x^2+9y^2)$ (B) $(2x-3y), (4x^2-9y^2)$
(C) $(2x+3y), (4x^2-6xy+9y^2)$ (D) $(2x-3y), (4x^2+6xy+9y^2)$

(iii) Factors of $3^{x^2} - x - 2$ are........
(A) $(x+1), (3x-2)$ (B) $(x+1), (3x+2)$
(C) $(x-1), (3x-2)$ (D) $(2x-3y), (4x^2+6xy)$

(iv) Factors of $a^4 - x - 2$ are ....
(A) $(a-b), (a+b), (a^2+4b^2)$ (B) $(a^2-2b^2), (a^2+2b^2)$
(C) $(a-b), (a+b), (a^2-4b^2)$ (D) $(a-2b), (a^2+2b^2)$

(v) What will be added to complete the square of $9a^2 - 12ab$?
(A) $-16b^2$ (B) $16b^2$ (C) $4b^2$ (D) $-4b^2$

(vi) Find m so that $x^2 + 4x + m$ is a complete square...

(viii) Factors of $27x^3 - \frac{1}{x^3}$

(A) $\left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$ (B) $\left(3x + \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$

(C) $\left(3x - \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$ (D) $\left(3x + \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$

(ix) x + 5x + 6 = ...

(A) $(x+2)(x-3)$ (B) $(x+2)(x+3)$

(C) $(x-2)(x+3)$ (D) $(x-2)(x-3)$

(x) $4a^2$ - 16 = ...

(A) $4(a+2)(a+2)$ (B) $4(a+2)(a+3)$

(C) $(a-2)(a+3)$ (D) $4(a-2)(a+2)$

(xi) $4a^2$ + 4ab + (........) is a complete square

(A) b^2 (B) $16b^2$ (C) $4b^2$ (D) $-4b^2$

(xii) $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$ = ...

(A) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$ (B) $\left(\frac{x}{y} + \frac{y}{x}\right)^2$ (C) $\left(\frac{y}{x}\right)^2$ (D) $\left(\frac{x}{y}\right)^2$

(xiii) $(x+y)(x^2 - xy + y^2)$ = ...

(A) $-x^3 - y^3$ (B) $-x^3 + y^3$ (C) $x^3 + y^3$ (D) $x^3 - y^3$

(xiv) Factored form of $x^4 - 16$ is ....

(A) $(x-2)(x-2)(x^2-4)$ (B) $(x-2)(x+2)(x^2+4)$

(C) $(x+2)(x+2)(x^2+4)$ (D) $(x-2)(x+2)(x^2-4)$

(xv) If $x - 2$ is factor of $p(x) = x^2 + 2kx + 8$, then k = ...

(A) -4 (B) 16 (C) 0 (D) -3

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Factorize (i) $144a^2 + 24a + 1$; (iii) $(x+y)^2 - 14z(x+y) + 49z^2$; EX #5.1 Q.3;(i , iii)

(ii) Factorize $\left(5x - \frac{1}{x}\right)^2 + 4\left(5x - \frac{1}{x}\right) + 4$; EX #5.2 Q.3;(viii)

(iii) Factorize (i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$; EX #5.2 Q.4;(i)

(iv) Factorize $(x+2)(x+3)(x+4)(x+5) - 15$; EX #5.2 Q.4;(iii)

(v) Factorize $(x+1)(x+2)(x+3)(x+6) - 3x^2$; EX #5.2 Q.4;(v)

(vi) Factorize (i) $x^3 + 48x - 12x^2 - 64$; (ii) $8x^3 + 60x^2 + 150x + 125$; EX #5.2 Q.5;(i , ii)

(vii) Factorize (i) $27 + 8x^3$; (ii) $125x^3 - 216y^3$; EX #5.2 Q.6;(i , ii)

(viii) If $(x+2)$ is a factor of $x^2 - 4kx - 4k^2$, then find the value(s) of k. ; EX #5.3 Q.2;(i)

(ix) Without actual long division determine whether $(x-2)$ and $(x-3)$ are factors of $p(x) = x^3 - 12x^2 + 44x - 48$; EX #5.3 Q.3;(i)

(x) Determine the value of k if $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$; EX #5.3 Q.5

(xi) The polynomial $x^3 + lx^2 + mx + 24$ has factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$. Find the values of l and m. ; EX #5.3 Q.7

(xii) The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible $x^2 - 5x + 6$. Find the values of a and

(xiv) Use Remainder theorem to find the remainder when $6x^4+2x^3-x+2$ is divided by $(x+2)$; EX #5.3 Q.1;(iii)

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3\times 8=24)$

Q.3 Show that the diagonals of the parallelogram having vertices $A(1,2)$, $B(4,2)$, $C(-1,-3)$ and $D(-4,-3)$ bisect each other. ; EX #9.3 ; Q.5

Q.4 The sum of lengths of any two sides of a triangle is greater than the length of the third side. ; Theorem # 13.1.3

Q.5 Find k, given that the point $(2,k)$ is equidistant from $(3,7)$ and $(9,1)$. ; EX #9.2 ; Q.5

Q.6 The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle. ; Theorem # 14.1.3

Q.7 Construct ΔABC such that $m\overline{AB}=3cm$, $m\overline{BC}=3.8\,cm$, $m\overline{AC}=4.8\,cm$.
Construct a rectangle equal in area to the ΔABC, and measure its sides. ; EX #17.4 Q.3

SOLUTION OF GUESS PAPER & MODEL PAPER # 5
(Reduced Syllabus)

SECTION- A (MCQs)

i. B	ii. C	iii. D	iv. B	v. C	vi. C
vii. C	viii. A	ix. B	x. D	xi. A	xii. A
xiii. C	xiv. B	xv. D			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9\times 4=36)$

(i) Factorize (i) $144a^2+24a+1$; (iii) $(x+y)^2-14z(x+y)+49z^2$; EX #5.1 Q.3;(i , iii)

(i) $144a^2+24a+1$

$=144a^2+12a+12a+1 \quad =12a(12a+1)+1(12a+1)$

$=(12a+1)\,(12a+1)=(12a+1)^2$

(iii) $(x+y)^2-14z(x+y)+49z^2$

$=(x+y)^2-2(x+y)(7z)+(7z)^2 \qquad =(x+y-7z)^2$

(ii) Factorize $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$; EX #5.2 Q.3;(viii)

Solution: Let $5x-\dfrac{1}{x}=y$

$=y^2+4y+4$

$=(y+2)^2=(y+2)(y+2)$

By putting value of $y=5x-\frac{1}{x}$

$=\left(5x-\dfrac{1}{x}+2\right)\left(5x-\dfrac{1}{x}+2\right)$

(iii) Factorize (i) $(x^2+5x+4)(x^2+5x+6)-3$; EX #5.2 Q.4;(i)

Solution: Let $x^2+5x=y$

$(y+4)\,(y+6)-3 \quad =y^2+6y+4y+24-3 \qquad =y^2+10y+21$

$=y^2+7y+3y+21=y(y+7)+3(y+7)$

$=(y+7)(y+3)$

(iv) Factorize $(x+2)(x+3)(x+4)(x+5)-15$; EX #5.2 Q.4;(iii)

Solution: By using commutative property of addition As $2+5=3+4$

$=(x+2)(x+5)(x+3)(x+4)-15$

$=(x^2+7x+10)(x^2+7x+12)-15$

Let $x^2+7x=y$

$=(y+10)(y+12)-15$ $\quad=y^2+22y+120-15$

$=y^2+22y+120-15$ $\quad=y^2+22y+105$

$=y^2+15y+7y+105$ $\quad=y(y+15)+7(y+15)$

$=(y+15)(y+7)$

By putting value of $y=x^2+7x$

$=(x^2+7x+15)(x^2+7x+7)$

(v) Factorize $(x+1)(x+2)(x+3)(x+6)-3x^2$; EX #5.2 Q.4;(v)

Solution: By using commutative property of multiplication

As $1\times 6=2\times 3$

$=(x+1)(x+6)(x+2)(x+3)-3x^2$

$=(x^2+7x+6)(x^2+5x+6)-3x^2=(x^2+6+7x)(x^2+6+5x)-3x^2$

Let $x^2+6=y$

$=(y+7x)(y+5x)-3x^2$

$=y^2+5xy+7xy+35x^2-3x^2$ $\quad=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$ $\quad=y(y+8x)+4x(y+8x)$

$=(y+8x)(y+4x)$

By putting value of $y=x^2+6$

$=(x^2+6+8x)(x^2+6+4x)$

$=(x^2+8x+6)(x^2+4x+6)=x\left(x+8+\frac{6}{x}\right)\cdot x\left(x+4+\frac{6}{x}\right)$

$=x^2\left(x+\frac{6}{x}+8\right)\left(x+\frac{6}{x}+4\right)$

(vi) Factorize (i) $x^3+48x-12x^2-64$; (ii) $8x^3+60x^2+150x+125$; EX #5.2 Q.5;(i , ii)

Solution: (i) $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$ $\quad=x^3-3.x^2.4+3.x.4^2-4^3=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution: $=(2x)^3+3.(2x)^2.5+3.(2x).5^2+5^3$ $\quad=(2x-5y)^3$

(vii) Factorize (i) $27+8x^3$; (ii) $125x^3-216y^3$; EX #5.2 Q.6;(i , ii)

Solution: (i) $27+8x^3$

$=(3)^2+(2x)^3=(3+2x)[3^2-3.2x+2(x)^2]$ $\quad=(3+2x)(9-6x+4x^2)$

(ii) $125x^3-216y^3$

Solution: $=(5x)^3-(6y)^3=(5x-6y)[(5x)^2+5x.6y+(6y)^2]$ $\quad=(5x-6y)(25x^2+30xy+36y^2)$

(viii) If $(x+2)$ is a factor of $x^2-4kx-4k^2$, then find the value(s) of k. ; EX #5.3 Q.2;(i)

Solution: Let $p(x)=3x^2-4kx-4k^2$

As $x+2=x-(-2)$ is a factor of $p(x)$

So $p(-2)=0$

$3(-2)^2-4k(-2)-4k^2=0$ $\Rightarrow$ $12+8k-4k^2=0$

Or $3+2k-k^2=0$ $\Rightarrow$ $3+3k-k-k^2=0$

$3(1+k)-k(1+k)=0$ $\Rightarrow$ $(1+k)(3-k)=0$

$1+k=0$; $3-k=0$

$k=-1$; $k=3$

The remainder for $x-2$ is
$p(2)=(2)^3-12(2)^2+44(2)-48 \quad =8-48+88-48=0$
Since remainder = 0, therefore $x-2$ is a factor of $p(x)$
The remainder for $x-3$ is
$p(3)=(3)^3+2(4)^2+44(3)-48 \quad =27-108+132-48 \ =3\neq 0$
Since remainder $\neq 0$, therefore $x-3$ is not a factor of $p(x)$

(x) Determine the value of k if $p(x)=kx^3+4x^2+3x-4$ and $q(x)=x^3-4x+k$ leaves the same remainder when divided by$(x-3)$; EX #5.3 Q.5

Solution: $p(x)=kx^3+4x^2+3x-4$
When $p(x)$ is divided by $x-3$, then the remainder $p(3)=0$
$p(3)=k(3)^3+4(3)^2+3(3)-4 \ =27k+36+9-4 \ =27k+41$
$q(x) \ =x^3-4x+k$
When $q(x)$ is divided by $x-3$ then the remainder $q(3)=0$
$q(3)=(3)^2-4(3)+k \ =27-12+k=15+k$
According to given condition $\quad p(3)=q(3)$
$27k+41=15+k \ \Rightarrow \quad 26k=-26 \quad \Rightarrow \quad k=-1$

(xi) The polynomial $x^3+lx^2+mx+24$ has factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$. Find the values of l and m. ; EX #5.3 Q.7

Solution: Let $p(x)=x^3+lx^2+mx+24$
As $x+4$ is a factor of $p(x)$; i.e. $(-4)^3+l(-4)^2+m(-4)+24=0$
$-64+16l-4m+24=0$
or $16l-4m=40$ or $4l-m=10$ .... (i)
When $p(x)$ is divided by $x-2$ When the remainder is $p(2)$ Then $p(2)=36$
$x^3+lx^2+mx+24 \ = \ 36 \ \Rightarrow \quad (2)^3+l(2)^2+m(2)+24=36$
$8+4l+2m+24=36 \ \Rightarrow \quad 4l+2m=4$
or $2l+3m=2$ .... (ii)
By adding eq. (i) and eq. (ii), we get $\quad 6l=12 \ \Rightarrow \ l=2$
By putting $l=2$ in eq. (i), we get; $\quad 8-m=10$
$-m=2 \Rightarrow \quad m=-2 \Rightarrow \quad$ So $\ l=2$, $m=-2$

(xii) The expression $ax^3-9x^2+bx+3a$ is exactly divisible x^2-5x+6. Find the values of a and b. ; EX #5.3 Q.9

Solution: Let $p(x)=ax^3-9x^2+bx+3a$ and $q(x)=x^2-5x+6$
$=x^2-3x-2x+6 \quad =x(x-3)-2(x-3) \quad =(x-3)(x-2)$
As $p(x)$ is exactly divisible by $q(x)$. So $p(x)$ is exactly divisible by $x-2$ and $x-3$
[$\because x=2$ and $x=3$] ; Hence $\quad p(2)=0$ And $\quad p(3)=0$
$p(2)=2(2)^3-9(2)^2+b(2)+3a=0$
$8a-36+2b+3a=0 \ \Rightarrow \quad 11a+2b=36$
Now $p(3)=a(3)^3-9(3)^2+b(3)+3a=0 \ \Rightarrow \quad 27a-81+3b+3a=0$
$30a+3b=81 \quad \Rightarrow \quad 10a+b=27$
By multiplying eq. (ii) by 2 and subtract from eq. (i), we get
$11a+2b=36$
$\underline{\pm 20a\pm 2b=\pm 54}$

(xiii) Use Remainder theorem to find the remainder when $3x^3 - 10x^2 + 13x - 6$ is divided by $(x-2)$; EX #5.3 Q.1;(i)

Solution: Let $p(x) = 3x^3 - 10x^2 + 13x - 6$

When $p(x)$ is divided by $x - 2$; The remainder $R = p(2)$

$p(2) = 3(2)^3 - 10(2)^2 + 13(2) - 6$

$p(2) = 24 - 40 + 26 - 6 \quad = 4$; Therefore remainder = 4

(xiv) Use Remainder theorem to find the remainder when $6x^4 + 2x^3 - x + 2$ is divided by $(x+2)$; EX #5.3 Q.1;(iii)

Solution: Let $p(x) = 6x^4 + 2x^3 - x + 2$

When $p(x)$ is divided by $x + 2$; The remainder $R = p(-2)$

$p(-2) = 6(-2)^4 + 2(-2)^3 - 2 + 2 \quad = 96 - 16 + 2 + 2 \quad = 84$

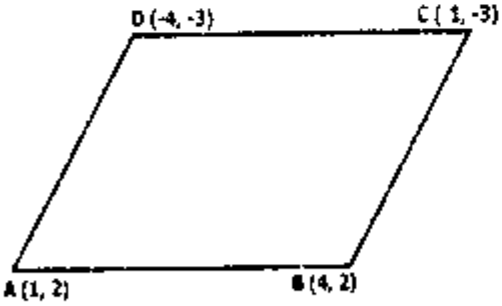
Therefore remainder = 84

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Show that the diagonals of the parallelogram having vertices $A(1,2)$, $B(4,2)$, $C(-1,-3)$ and $D(-4,-3)$ bisect each other. ; EX #9.3 ; Q.5

Solution:



M_1 mid point of AC is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{1-1}{2}, \frac{2-3}{2}\right)$

Or $\left(\frac{0}{2}, \frac{-1}{2}\right)$ Or $M_1\left(0, -\frac{1}{2}\right)$

M_2 mid point of BD is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-4+4}{2}, \frac{-3+2}{2}\right)$

or $\left(\frac{0}{2}, -\frac{1}{2}\right)$ Or $M_1\left(0, -\frac{1}{2}\right)$

Since both the diagonals have same midpoint therefore they bisect each other.

Q.4 The sum of lengths of any two sides of a triangle is greater than the length of the third side. ; Theorem # 13.1.3

Solution:

Given:

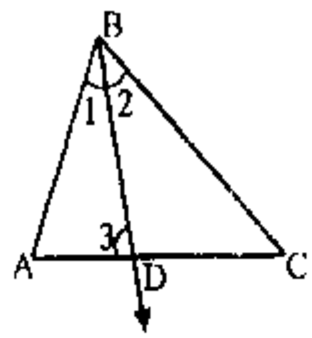
A triangle ABC

To prove:

(i) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

(ii) $m\overline{AC} + m\overline{AB} > m\overline{BC}$

(iii) $m\overline{AC} + m\overline{BC} > m\overline{AB}$



Construction: Draw the bisector of $\angle B$ to meet the side $\overline{AC}$ at the point D.

Proof:

Statements	Reasons
In ΔCBD	
$m\angle 3 > m\angle 2$ .... (i)	Exterior angle is greater than non adjacent interior angle
$m\angle 2 = m\angle 1$ .... (ii)	Construction
$\therefore \quad m\angle 3 > m\angle 1$	By (i) and(ii)

$m\overline{AB} + m\overline{BC} > m\overline{AD} + m\overline{DC}$ or $m\overline{AB} + m\overline{BC} > m\overline{AC}$ Similarly by drawing angle bisector of $\angle A$ and $\angle C$ it canbe proved that $m\overline{AC} + m\overline{AB} > m\overline{BC}$ and $m\overline{AC} + m\overline{BC} > m\overline{AB}$	Adding (iii) and (iv) $\therefore \quad m\overline{AD} + m\overline{DC} = m\overline{AC}$

Q.5 Find k, given that the point $(2, k)$ is equidistant from $(3, 7)$ and $(9, 1)$. ; EX #9.2 ; Q.5

Solution: Let the given points be $P(2, k)$ and $A(3,\ 7)$, $B(9,\ 1)$.

As the points P is equidistant from A and B.

$\therefore \quad |PA| = |PB|$

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

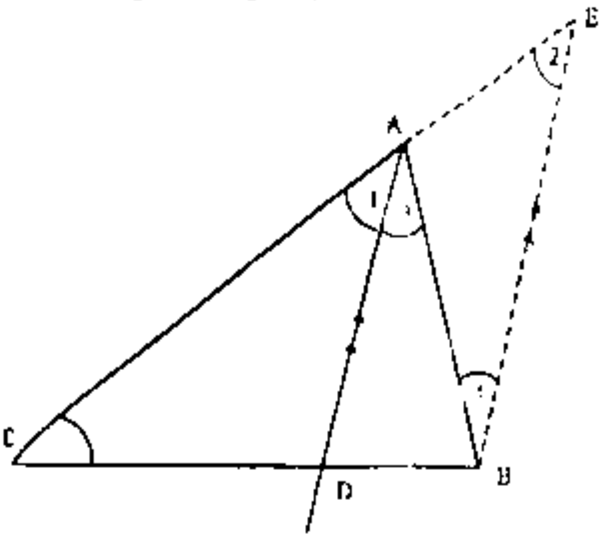
i.e. $\quad \sqrt{(2-3)^2 + (k-7)^2} = \sqrt{(2-9) + (k-1)}$

Squaring both sides we have $\quad (-1) + (5-7)^2 = (-7)^2 + (k-1)^2$

$$1 + k^2 - 14k + 49 = 49 + k^2 - 2k + 1 \Rightarrow \quad 50 + k^2 - 14k = 50 + k^2 - 2k$$

$$-14 + 2k = 0 \Rightarrow \quad -12k = 0 \Rightarrow \quad k = 0$$

Q.6 The internal bisector of an angle of a triangle divides the sides opposite to it in the ratio of the lengths of the sides containing the angle. ; Theorem # 14.1.3



Given: In ΔABC internal angle bisector of angle A intersects $\overline{CB}$ at the point D.

To Prove: $m\overline{BD} : m\overline{DC} = m\overline{AB} : m\overline{AC}$

Construction: Draw a line segment $\overline{BE} \| \overline{DA}$ to meet $\overline{CA}$ produced, at E.

Proof:

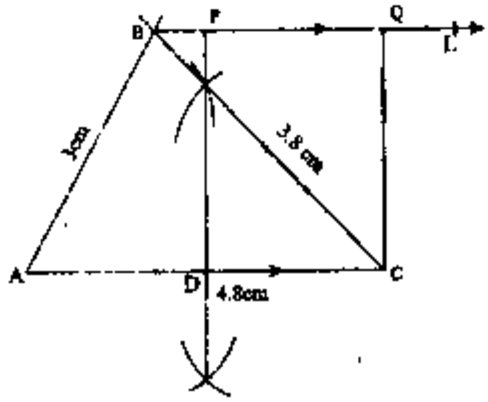
Statements	Reasons
$\because \overline{AD} \| \overline{EB}$ and $\overline{EC}$ intersects there at A	Constructions
and E, so $m\angle 1 = m\angle 2$	Corresponding angles
Again AD ‖ EB	
And AB intersects them	
So $m\angle 3 = m\angle 4$ .... (ii)	Alternate angles
But $m\angle 1 = m\angle 3$	Construction (given)
$\therefore m\angle 2 = m\angle 4$	

$\therefore \quad \dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$ or $\dfrac{m\overline{BD}}{m\overline{DC}} = \dfrac{m\overline{EA}}{m\overline{AC}}$ $m\overline{BD}:m\overline{DC} = m\overline{AB}:m\overline{AC}$	A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally. $\therefore m\overline{EA} = m\overline{AB}$ (Proved)

Q.7 Construct ΔABC such that $m\overline{AB} = 3cm$, $m\overline{BC} = 3.8\,cm$, $m\overline{AC} = 4.8\,cm$.
Construct a rectangle equal in area to the ΔABC, and measure its sides. ; EX #17.4 Q.3

Solution:

Construction:



(i) Draw a line segment $m\overline{AC} = 4.8\,cm$.
(ii) With centre at A and radius 3 cm draw an arc.
(iii) With centre at C and radius 3.8 cm draw another arc to cut the first arc at B.
(iv) Join $\overline{AB}$ and $\overline{BC}$ to complete the ΔABC.
(v) Draw $\overline{BL} \parallel \overline{AC}$.
(vi) Draw $\overline{DP}$ the perpendicular bisector of $\overline{AC}$ to meet $\overline{BL}$ at P.
(vii) Cut off $m\overline{PQ} = m\overline{DC}$.
(viii) Join Q to C.
(ix) Then PQCD is the required rectangle.
(x) Measure the sides of the rectangle, $m\overline{DC} = 2.4\,cm$ and $m\overline{DP} = 2.3\,cm$

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Factorize (i) $2abc - 4abx + 2abd$; **EX #5.1 Q.1;(i , v, vi)**

$2abc - 4abx + 2abd = 2ab(c - 2x + d)$

(v) $3x^3y(x-3y) - (7x^2y^2(x-3y)$

$= (x-3y)(3x^3y - 7x^2y^2) \qquad = (x-3y)\,x^2y(3x-7y) = x^2y(x-3y)x^2y(3x-7y)$

(vi) $2xy^3(x^2+5) + 8xy^2(x^2+5)$

$= (x^2+5)(2xy^3 + 8xy^2) = (x^2+5)(2xy^2)(y+4) = 2xy^2(x^2+5)(y+4)$

Q2.Factorize $x^3 + 3xy^2 - 2x^2 - 6y^3$; **EX #5.1 Q.2;(iii , iv)**

(iii) $x^3 + 3xy^2 - 2x^2 - 6y^3 \qquad = x(x^2+3y^2) - 2y(x^2+3y^2) \qquad = (x^2+3y^2)(x-2y)$

(iv) $(x^2 - y^2)z + (y^2 - z^2)x \qquad = x^2z - y^2z + y^2x - z^2x \qquad = x^2z - z^2x + y^2x - y^2z$

$= xz(x-z) + y^2x - y^2z \qquad = (x-z)(xz + y^2)$

Q4.Factorize **EX #5.1 Q.4;(ii , iv)**

(ii) $x(x-1) - y(y-1) \quad = x^2 - x - y^2 + y \quad = x^2 - y^2 - x + y$

$= (x+y)(x-y) - 1(x-y) = (x-y)(x+y-1)$

(iv) $3x - 243x^3$

$= 3x(1 - 81x^2) \qquad = 3x\{(1)^2 - (9x)^2\} \qquad = 3x(1+9x)(1-9x)$

Q5. **Factorize EX #5.1 Q.5;(i , ii, iii)**

(ii) $x^2 - a^2 + 2a - 1$
$= x^2 - (a^2 - 2a + 1)$ $= x^2 - [(a)^2 - 2(a)(1) + (1)^2]$ $= x^2 - (a-1)^2$
$= (x)^2 - (a-1)^2$ $= [x + (a-1)][x - (a+1)]$ $= (x + a - 1)(x - a + 1)$

(iii) $4x^2 - y^2 - 4x - 2y + 3$
$= 4x^2 - (y^2 + 2y + 1)$ $= (2x)^2 - (y+1)^2$
$= [2x + (y+1)][2x - (y+1)]$ $= (2x + y - 1)(2x - y - 1)$

Q1. Factorize EX #5.2 Q.1;(i , iv, v)

(i) $x^4 + \frac{1}{x^4} - 3$

Solution: $= x^4 + \frac{1}{x^4} - 2 - 1 = x^4 - 2 + \frac{1}{x^4} - 1$

$= \left(x^2 - \frac{1}{x^2}\right)^2 - 1^2$ $= \left[\left(x^2 - \frac{1}{x^2}\right) + 1\right]\left[\left(x^2 - \frac{1}{x^2}\right) - 1\right]$ $= \left(x^2 - \frac{1}{x^2} + 1\right)\left(x^2 - \frac{1}{x^2} - 1\right)$

(iv) $4x^4 + 81$

Solution: $= (2x^2)^2 + (9)^2 + 36x - 36x^2$ $= (2x^2 + 9)^2 - (6x)^2$
$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$ $= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$

(v) $x^4 + x^2 + 25$

Solution: $= x^4 + 10x^2 + 25 - 9x^2$
$= (x^2)^2 + 2(x^2)5 - 9x^2$ $= (x^2 + 5)^2 - (3x)^2$
$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$ $= (x^2 + 2x + 4)(x^2 - 2x + 4)$

Factorize (i) $x^2 + 14x + 48$; EX #5.2 Q.2;(i , iii)

Solution: $= x^2 + 8x + 6x + 48$ $= x(x+8) + 6(x+8) = (x+8)(x+6)$

(iii) $x^2 - 11x - 42$

Solution: $= x^2 - 14x + 3x - 42$ $= x(x-14) + 3(x-14)$ $= (x-14)(x+3)$

Q3.Factorize EX #5.2 Q.3;(ii , v)

(ii) $30x^2 + 7x - 15$

Solution: $30x^2 + 25x - 18x - 15$ $= 5x(6x+5) - 3(6x+5)$ $= (6x+5)(5x-3)$

(v) $4x^2 - 17xy + 4y^2$

Solution: $4x^2 - 16xy - xy + 4y^2$ $= 4x(x-4y) - y(x-4y)$ $= (x-4y)(4x-y)$

Q1. Multiple choice questions. Choose the correct answer. Review EX #5 Q.1

(i) The factors of $x^2 - 5x + 6$ are.....
(a) $x+1, x-6$ (b) $x-2, x-3$
(c) $x+6, x-1$ (d) $x+2, x+3$

(ii) Factors of $8x^3 + 27y^3$ are.......
(a) $(2x+3y), (4x^2+9y^2)$ (b) $(2x-3y), (4x^2-9y^2)$
(c) $(2x+3y), (4x^2-6xy+9y^2)$ (d) $(2x-3y), (4x^2+6xy+9y^2)$

(iii) Factors of $3^{x^2} - x - 2$ are........
(a) $(x+1), (3x-2)$ (b) $(x+1), (3x+2)$
(c) $(x-1), (3x-2)$ (d) $(2x-3y), (4x^2+6xy)$

(iv) Factors of $a^4 - x - 2$ are ....
(a) $(a-b), (a+b), (a^2+4b^2)$ (b) $(a^2-2b^2), (a^2+2b^2)$
(c) $(a-b), (a+b), (a^2-4b^2)$ (d) $(a-2b), (a^2+2b^2)$

(vi) Find m so that $x^2 + 4x + m$ is a complete square...
(a) 8 (b) −8 (c) 4 (d) 16

(vii) Factors of $5x^2 - 17xy - 12y^2$ are.......
(a) (x + 4y), (5x + 3y) (b) (x − 4y), (5x − 3y)
(c) (x − 4y), (5x + 3y) (d) (5x − 4y), (x + 3y)

(viii) Factors of $27x^3 - \frac{1}{x^3}$
(a) $\left(3x - \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$ (b) $\left(3x + \frac{1}{x}\right), \left(9x^2 + 3 + \frac{1}{x^2}\right)$
(c) $\left(3x - \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$ (d) $\left(3x + \frac{1}{x}\right), \left(9x^2 - 3 + \frac{1}{x^2}\right)$

Answers:

(i) b	(ii) c	(iii) d	(iv) b
(v) c	(vi) c	(vii) c	(viii) a

Review EX #5 Q.2

Q2. Complete items. Fill in the blanks.
(i) x + 5x + 6 = ...
(ii) $4a^2$ - 16 = ...
(iii) $4a^2$ + 4ab + (.........) is a complete square
(iv) $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$ = ...s
(v) $(x + y)(x^2 - xy + y^2)$ = ...
(vi) Factored form of $x^4 - 16$ is ....
(vii) If $x - 2$ is factor of $p(x) = x^2 + 2kx + 8$, then k = ...

Answers:

(i) $(x + 2)(x + 3)$ (ii) $4(a - 2)(a + 2)$
(iii) b^2 (iv) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$
(v) $x^3 + y^3$ (vi) $(x - 2)(x + 2)(x^2 + 4)$
(vii) −3

GUESS PAPER & MODEL PAPER # 06
BASED ON UNIT # 6 (Reduced Syllabus)
ALGEBRAIC MANIPULATION

Unit 6	Algebraic Manipulation
Exercise 6.1	Q1; Q2(i, ii, iii); Q3(i, iii); Q4; Q5(ii, iii); Q6; Q8; Q9
Exercise 6.2	Q1; Q2; Q4; Q6; Q9; Q11; Q13
Exercise 6.3	Q1(i, iv, vi, vii); Q2(i, iv, v); Q3(i); Q4(i)
Review Ex 6	Q1; Q8

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) H.C.F. of $p^3q - pq^3$ and $p^5q^2 - p^2q^5$ is......
(A) $pq(p^2 - q^2)$ (B) $pq(p - q)$ (C) $p^2q^2(p - q)$ (D) $pq(p^3 - q^3)$

(ii) H.C.F. of $5x^2y^2$ and $20x^3y^3$ is......
(A) $5x^2y^2$ (B) $20x^3y^3$ (C) $100x^5y^5$ (D) $5xy$

(iii) H.C.F. of $x - 2$ and $x^2 + x - 6$ is.......
(A) $x^2 + x - 6$ (B) $x + 3$ (C) $x - 2$ (D) $x + 6$

(iv) H.C.F. of $a^3 + b^3$ and $a^2 - ab + b^2$ is......
(A) $a + b$ (B) $a^2 - ab + b^2$ (C) $(a - b)^2$ (D) $a^2 + b^2$

(v) H.C.F. of $x^2 - 5x + 6$ and $x^2 - x - 6$ is.......
(A) $x - 3$ (B) $x + 2$ (C) $x^2 - 4$ (D) $x - 2$

(vi) H.C.F. of $a^2 - b^2$ and $a^3 - b^3$ is......
(A) $a - b$ (B) $a + b$ (C) $a^2 + ab + b^2$ (D) $a^2 - ab + b^2$

(vii) H.C.F. of $x^2 + 3x + 2$, $x^2 + 4x + 3$ and $x^2 + 5x + 4$ is......
(A) $x + 1$ (B) $(x + 1)(x + 2)$ (C) $x + 3$ (D) $(x + 4)(x + 1)$

(viii) L.C.M. of $15x^2$, $45xy$ and $30xyz$ is ....
(A) $90xyz$ (B) $90x^2yz$ (C) $15xyz$ (D) $15x^2yz$

(ix) L.C.M. of $a^2 + b^2$ and $a^4 - b^4$ is ......

(xi) Simplify $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}=\cdots$

(A) $\frac{4a}{9a^2-b^2}$ (B) $\frac{4a-b}{9a^2-b^2}$ (C) $\frac{4a+b}{9a^2-b^2}$ (D) $\frac{b}{9a^2-b^2}$

(xii) Simplify $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=\cdots$

(A) $\frac{a+7}{a-6}$ (B) $\frac{a+7}{a-2}$ (C) $\frac{a+3}{a-6}$ (D) $\frac{a-2}{a+3}$

(xiii) Simplify $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}=\cdots$

(A) $\frac{1}{a+b}$ (B) $\frac{1}{a-b}$ (C) $\frac{a-b}{a^2+b^2}$ (D) $\frac{a+b}{a^2+b^2}$

(xiv) Simplify $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=\cdots$

(A) $\frac{x}{x+y}$ (B) $\frac{y}{x+y}$ (C) $\frac{y}{x}$ (D) $\frac{x}{y}$

(xv) The square root of a^2-2a+1 is......

(A) $\pm(a+1)$ (B) $\pm(a-1)$ (C) $a-1$ (D) $a+1$

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9\times 4=36)$

(i) For what value of k is $(x+4)$ the H.C.F of $(x^2+x-(2k+2)$ and $2x^2+kx-12$?

EX #6.1 Q.6

(ii) Simplify each of the following as a rational expression. $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

; EX #6.2 Q.3

(iii) Simplify each of the following as a rational expression. $A-\frac{1}{A}$ where $A=\frac{a+1}{a-1}$

EX #6.2 Q.6

(iv) Perform the indicated operations and simplify to the lowest forms. $\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

EX #6.2 Q.9

(v) Perform the indicated operations and simplify to the lowest forms.
$\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$; EX #6.2 Q.13

(vi) Find the value of k for which the following expressions will become a perfect square.
$4x^4-12x^3+37x^2-42x+k$; EX #6.3 Q.3;(i)

(vii) Find square root by using division method. $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}\ (x\neq 0)(y\neq 0)$

Review EX #6 Q.8

(viii) Find the value of l and m for which the following expressions will become a perfect squares. $x^4+4x^3+16x^2-lx+m$; EX #6.3 Q.4;(i)

(ix) Perform the indicated operations and simplify to the lowest forms.
$\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$; EX #6.2 Q.11

(xi) Simplify each of the following as a rational expression. $\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$
EX #6.2 Q.2

(xii) Let $p(x)=10(x^2-9)(x^2-3x+2)$ and $q(x)=10x(x+3)(x-1)^2$. If the H.C.F. of $p(x), q(x)$ is $10(x+3)(x-1)$, find their L.C.M. ; EX #6.1 Q.9

(xiii) The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $p(x)=x^3+x^2+x+1$, find $q(x)$; EX #6.1 Q.8

(xiv) Find the H.C.F. of the following by division method.
$x^3+3x^2-16x+12$, $x^3+x^2-10x+8$; EX #6.1 Q.3;(i)

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3\times 8=24)$

Q.3 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are collinear.
EX #9.2 ; Q.6

Q.4 From a point, outside a line, the perpendicular is the shortest distance from the point to the line. ; Theorem # 13.1.4

Q.5 Verify that the Δs having the following measures of sides are right – angled.
(i) $a=5cm,\ b=12cm,\ c=13$ (ii) $a=1.5\ cm, b=2cm, 2.5cm$
(iii) $a=9cm, b=12cm, c=15cm$ (iv) $a=16cm, b=30cm, c=34cm$
EX #15; Q.1

Q.6 A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport? ; EX #15; Q.7

Q.7 Construct the following Δ's XYZ. Draw their three medians and show that they are concurrent? $m\overline{XY}=4.5$ cm, $m\overline{YZ}=3.4$ cm, $m\overline{ZX}=5.6$; EX #17.2 Q.4;(ii)

SOLUTION OF GUESS PAPER & MODEL PAPER # 6
(Reduced Syllabus)

SECTION- A (MCQs)

i. B	ii. A	iii. C	iv. B	v. A	vi. A
vii. A	viii. B	ix. C	x. C	xi. C	xii. A
xiii. A	xiv. D	xv. B			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9\times 4=36)$

(i) For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?
EX #6.1 Q.6

Solution: Let $P(x)=x^2+x-(2k+2)$ And $q(x)=2x^2+kx-12$
As $x+4$ is H.C.F. of p(x) and q(x). So p(x)is exactly divisible by $x+4$ and thus $p(-4)=0$
i.e. $(-4)^2+(-4)-(2k+2)=0 \Rightarrow$ $16-4-2k-2=0$
$\Rightarrow$ $10-2k=0$ $\Rightarrow$ $2k=10 \Rightarrow$ $k=5$

$$= \frac{x^2-3x+2x-6}{x^2-3^2} + \frac{x^2+6x-4x-24}{x^2-4x+3x-12} \quad = \frac{x(x-3)+2(x-3)}{(x+3)(x-3)} + \frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$= \frac{(x-3)(x+2)}{(x+3)(x-3)} + \frac{(x+6)(x-4)}{(x-4)(x+3)} = \frac{x+2}{x+3} + \frac{x+6}{x+3} = \frac{x+2+x+6}{x+3} = \frac{2x+8}{x+3} = \frac{2(x+4)}{x+3}$$

(iii) Simplify each of the following as a rational expression. $A - \frac{1}{A}$, **where** $A = \frac{a+1}{a-1}$

EX #6.2 Q.6

Solution: $A - \frac{1}{A} = \frac{a+1}{a-1} - \frac{a-1}{a+1}$

$$= \frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)} = \frac{a^2+2a+1-(a^2-2a+1)}{a^2-1} \quad = \frac{a^2+2a+1-a^2+2a-1}{a^2-1} = \frac{4a}{a^2-1}$$

(iv) Perform the indicated operations and simplify to the lowest forms. $\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$

EX #6.2 Q.9

Solution: $\frac{x^2+x-6}{x^2-x-6} \times \frac{x^2-4}{x^2-9}$

$$= \frac{x^2+3x-2x-6}{x^2-3x+2x-6} \times \frac{x^2-2^2}{x^2-3^2} \quad = \frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)} \quad = \frac{(x+3)(x-2)}{(x-3)(x+2)} \times \frac{(x+2)(x-2)}{(x+3)(x-3)}$$

$$= \frac{(x+3)(x-2)}{(x-3)(x+2)} = \frac{(x-2)^2}{(x-3)^2}$$

(v) Perform the indicated operations and simplify to the lowest forms.

$\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$; **EX #6.2 Q.13**

Solution: $\left[\frac{x^2+y^2}{x^2-y^2} - \frac{x^2-y^2}{x^2+y^2}\right] \div \left[\frac{x+y}{x-y} - \frac{x-y}{x+y}\right]$

$$= \frac{(x^2+y^2)^2-(x^2-y^2)^2}{(x^2-y^2)(x^2+y^2)} \div \frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}$$

$$= \frac{x^4+2x^2y^2+y^4-(x^4-2x^2y^2+y^4)}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-(x^2-2xy+y^2)}{(x-y)(x+y)}$$

$$= \frac{x^4+2x^2y^2+y^4-x^4+2x^2y^2-y^4}{(x^2-y^2)(x^2+y^2)} \div \frac{x^2+2xy+y^2-x^2+2xy-y^2}{(x-y)(x+y)}$$

$$= \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \div \frac{4xy}{(x^2-y^2)} \quad = \frac{4x^2y^2}{(x^2-y^2)(x^2+y^2)} \times \frac{x^2-y^2}{4xy} = \frac{xy}{x^2+y^2}$$

(vi) Find the value of k **for which the following expressions will become a perfect square.**

$4x^4 - 12x^3 + 37x^2 - 42x + k$; **EX #6.3 Q.3;(i)**

Solution:

$$\begin{array}{r|l}
 & 2x^2 - 3x + 7 \\
\hline
2x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + k \\
 & \pm 4x^4 \\
\hline
84x^2 - 3x & -12x^3 + 37x^2 \\
 & \mp 12x^3 \pm 9x^2 \\
\hline
4x^2 - 6x + 7 & 28x^2 - 42x + k \\
 & \pm 28x^2 \mp 42x \pm 49 \\
\hline
 & k - 49
\end{array}$$

The given expression will be perfect square when remainder = 0

(vii) Find square root by using division method. $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}(x\neq 0)(y\neq 0)$

Review EX #6 Q.8

Solution: $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$

$$\begin{array}{r|l}
 & \frac{2x}{y}+5-\frac{3y}{x} \\ \hline
\frac{2x}{y} & \frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2} \\
 & \pm\frac{4x^2}{y^2} \\ \hline
\frac{4x}{y}+5 & \frac{20x}{y}+13 \\
 & \mp\frac{20x}{y}\pm 25 \\ \hline
\frac{4x}{y}+10-\frac{3y}{x} & -12-\frac{30y}{x}+\frac{9y^2}{x^2} \\
 & \pm 12\mp\frac{30y}{x}\pm\frac{9y^2}{x^2} \\ \hline
 & 0
\end{array}$$

So the required root is $\pm\left(\frac{2x}{y}+5-\frac{3y}{x}\right)$

(viii) Find the value of l and m for which the following expressions will become a perfect squares. $x^4+4x^3+16x^2-lx+m$; EX #6.3 Q.4;(i)

Solution: $x^4+4x^3+16x^2-lx+m$

$$\begin{array}{r|l}
 & x^2+2x+6 \\ \hline
x^2 & x^4+4x^3+16x^2+lx+m \\
 & \pm x^4 \\ \hline
2x^2+2x & 4x^3+16x^2+lx+m \\
 & \mp 4x^3\pm 4x^2 \\ \hline
2x^2+4x+6 & 12x^2+lx+m \\
 & \pm 12x^2\pm 24x\pm 36 \\ \hline
 & (l-24)x-(m-36)
\end{array}$$

The given expression will be perfect square when remainder = 0

if $l-24=0$

and $m-36=0$

$\therefore$ $l=24$, $m=36$.

(ix) Perform the indicated operations and simplify to the lowest forms.

$\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$; EX #6.2 Q.11

Solution: $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$ $=\frac{x(x-2)(x^2+2x+4)}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$

(x) Simplify each of the following as a rational expression. $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

EX #6.2 Q.4

Solution: $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

$=\frac{(x+2)(x+3)}{x^2-3^2}+\frac{(x+2).2(x^2-16)}{(x-4)(x^2-3x+2x-6)}$ $=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)\,(x+4)\,(x-4)}{(x-4)[x(x-3)+2(x-3)]}$

$=\frac{x+2}{x-3}+\frac{2(x+2)\,(x+4)}{(x-3)(x+2)}$ $=\frac{x+2}{x-3}+\frac{2\,(x+4)}{x-3}$ $=\frac{x+2+2x+8}{x-3}$ $=\frac{3x+10}{x-3}$

(xi) Simplify each of the following as a rational expression. $\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$

EX #6.2 Q.2

Solution: $\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$

$=\frac{(x+1)^2(x^2+1)-(x-1)^2(x^2+1)-4x(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}+\frac{4x}{x^4-1}$

$=\frac{(x^2+2x+1)\,(x^2+1)-(x^2-2x+1)(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}+\frac{4x}{x^4-1}$

$=\frac{x^4+x^2+2x^3+2x+1-(x^4+x^2-2x^3-2x+x^2+1)-(4x^3-4x)}{x^4-1}+\frac{4x}{x^4-1}$

$=\frac{x^4+2x^3+2x^2+2x+1-x^4+2x^3-2x^2+2x-1-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$ $=\frac{4x^3+4x-4x^3+4x}{x^4-1}+\frac{4x}{x^4-1}$

$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$ $=\frac{8x+4x}{x^4-1}$ $=\frac{12x}{x^4-1}$

(xii) Let $p(x)=10(x^2-9)(x^2-3x+2)$ and $q(x)=10x(x+3)(x-1)^2$. If the H.C.F. of $p(x),q(x)$ is $10(x+3)(x-1)$, find their L.C.M. ; EX #6.1 Q.9

Solution: $p(x)=10(x^2-9)(x^2-3x+2)$; $q(x)=10x(x+3)(x-1)^2$

$H.C.F.=10(x+3)(x-1)$

$$\text{L.C.M.}=\frac{p(x)\times q(x)}{H.C.F.}=\frac{10(x^2-9)(x^2-3x+2)10x(x+3)(x-1)^2}{10(x+3)(x-1)}$$

$=10(x^2-9)(x^2-3x+2)\,.\,x(x-1)$

$=10(x^2-9)[(x^2-2x-x+2)\,x.(x-1)]$

$=10(x^2-9)[x(x-2)-1(x-2].\,x.(x-1)$

$=10(x^2-9)(x-2)(x-1)\,.\,x\,.(x-1)$

$=10(x^2-9)(x-2)\,.\,x\,.\,(x-1)^2$

$=10x(x-2)(x-1)^2(x^2-9)$

(xiii) The L.C.M. and H.C.F. of two polynomials $p(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $p(x)=x^3+x^2+x+1$, find $q(x)$. ; EX #6.1 Q.8

Solution: $L.C.M.=2(x^4-1)$; $H.C.F.=(x+1)(x^2+1)$

$p(x)=x^3+x^2+x+1$

$q(x)=\frac{(L.C.M.)\times(H.C.F.)}{p(x)}$ $=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$ $=\frac{2(x^4-1)x^3+x+x^2+1}{x^3+x^2+x+1}$

(xiv) **Find the H.C.F. of the following by division method.**
$x^3 + 3x^2 - 16x + 12$, $x^3 + x^2 - 10x + 8$; **EX #6.1 Q.3;(i)**

Solution:

$$\begin{array}{r|l} & \quad 1 \\ x^3 + x^2 - 10x + 8 & x^3 + 3x^2 - 16x + 12 \\ & \underline{\pm x^3 \pm x^2 \mp 10x \pm 8} \\ & 2x^2 - 6x + 4 \\ & 2(x^2 - 3x + 2) \end{array}$$

By Ignoring 2

$$\begin{array}{r|l} & \quad x + 4 \\ x^2 - 3x + 2 & x^3 + x^2 - 10x + 8 \\ & \underline{\pm x^3 \mp 3x^2 \pm 2x} \\ & 4x^2 - 12x + 8 \\ & \underline{\pm 4x^2 \mp 12x \pm 8} \\ & \quad 0 \end{array}$$

$\text{H.C.F.} = x^2 - 3x + 2$

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. **(3 × 8 = 24)**

Q.3 Use distance formula to verify that the points $A(0,7), B(3,-5), C(-2,15)$ are collinear.
EX #9.2 ; Q.6

Solution: Let the points be A(0,7), B(3,5) and C(-2,15).

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(0-3)^2 + (7+5)^2} = \sqrt{(-3)^2 + (12)^2} = \sqrt{9+144} = \sqrt{153} = \sqrt{9 \times 17} = 3\sqrt{17}$

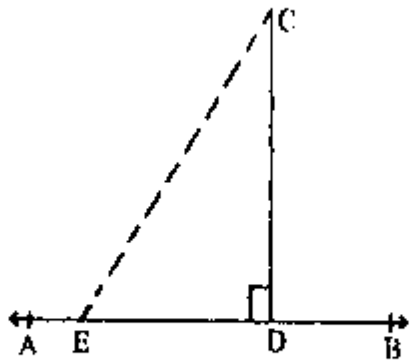
$|BC| = \sqrt{(-2-3)^2 + (15+5)^2} = \sqrt{(-5)^2 + (20)^2} = \sqrt{25+400} = \sqrt{425} = \sqrt{25 \times 17} = 5\sqrt{17}$

$|CA| = \sqrt{(0+2)^2 + (7-15)^2} = \sqrt{(2)^2 + (-8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$

By applying the condition of collinear points $|AB| + |CA| = 3\sqrt{17} + 2\sqrt{17} = (3+2)\sqrt{17}$
$= (3+2)\sqrt{17} = 5\sqrt{17} = |BC|$; $\therefore$ the given points are collinear.

Q.4 From a point, outside a line, the perpendicular is the shortest distance from the point to the line. **; Theorem # 13.1.4**

Solution:



Given:

A line $\overleftrightarrow{AB}$ and a point C (not lying on $\overleftrightarrow{AB}$) and a point D on $\overleftrightarrow{AB}$ such that $\overline{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

To prove:

$m\overline{CD}$ is the shortest distance from point C to the line $\overleftrightarrow{AB}$.

Construction:

Take a point E on $\overleftrightarrow{AB}$. Join C and E to get a Δ CDE.

Proof:

Statements	Reasons
In Δ CDE $m\angle CDB > m\angle CED$	An exterior angle of a triangle is greater than every non adjacent interior angle.
But $m\angle CDB = m\angle CDE$	Supplement of right angle.

BUT E was any point on AB Hence $m\overline{CD}$ is the shortest distance from C to $\overrightarrow{AB}$.	

Q.5 **Verify that the Δs having the following measures of sides are right – angled.**

(i) $a = 5cm,\ b = 12cm,\ c = 13$ (ii) $a = 1.5\ cm, b = 2cm, 2.5cm$
(iii) $a = 9cm, b = 12cm, c = 15cm$ (iv) $a = 16cm, b = 30cm, c = 34cm$

EX #15; Q.1

Solution: (i) $a = 5cm,\ b = 12cm,\ c = 13$

By Pythagoras theorem $a^2 + b^2 = (5)^2 + (12)^2$
$= 25 + 144 = 169 \Rightarrow c^2 = (13)^2 = 169 \Rightarrow \therefore a^2 + b^2 = c^2$
Thus the triangle is right angled triangle.

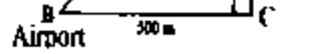
(ii) $a = 1.5\ cm, b = 2cm, 2.5cm$
Solution: By Pythagoras theorem $a^2 + b^2 = (1.5) + (2)$
$= 2.25 + 4 = 6.25 \Rightarrow c^2 = (2.5)^2 = 6.25 \Rightarrow a^2 + b^2 = c^2$
Thus the triangle is right angled triangle.

(iii) $a = 9cm, b = 12cm, c = 15cm$
Solution: By Pythagoras theorem $a^2 + b^2 = (9)^2 + (12)^2$
$= 81 + 144 = 225 \Rightarrow c^2 = (15) = 225 \Rightarrow \therefore a^2 + b^2 = c^2$
Hence the triangle is right angled triangle.

(iv) $a = 16cm, b = 30cm, c = 34cm$
Solution: By Pythagoras theorem $a^2 + b^2 = (16)^2 + (2)^2$
$= 256 + 900 = 1156 \Rightarrow c^2 = (34)^2 = 1156 \Rightarrow \therefore a^2 + b^2 = c^2$
Hence the triangle is right angled triangle.

Q.6 **A plane is at a height of 300 m and is 500 m away from the airport as shown in the figure. How much distance will it travel to land at the airport? ; EX #15; Q.7**

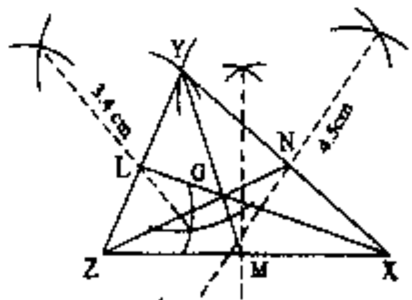
Solution:
$m\overline{BC} = 500\ m$; $m\overline{AC} = 300\ m$
By Pythagoras theorem $m\overline{AB}^2 = m\overline{BC}^2 + m\overline{AC}^2$
$m\overline{AB}^2 = (500)^2 + (300)^2$
$= 250000 + 90000$
$= 340000 = \sqrt{34 \times 10000}$
$m\overline{AB} = 100\sqrt{34}\ m$



Q.7 **Construct the following $\Delta's$ XYZ. Draw their three medians and show that they are concurrent? $m\overline{XY}$ = 4.5 cm, $m\overline{YZ}$ = 3.4 cm, $m\overline{ZX}$ = 5.6 ; EX #17.2 Q.4;(ii)**

Solution: Construction:

(i) Take $m\overline{ZX}$ = 5.6 cm.
(ii) With centre Z and radius $m\overline{ZY}$ = 3.4 cm draw an arc.
(iii) With centre X and radius $m\overline{XY}$ = 4.5 cm which intersects the first arc at Y.
(iv) Join $\overline{ZY}$ and $\overline{XY}$ to get the ΔXYZ.
(v) Draw the perpendicular bisectors of the sides $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$ of the ΔXYZ and mark their mid points L, M and N



(viii) The medians $\overline{XL}$ and $\overline{YM}$ meet in the point G.
(ix) Now draw the third median $\overline{ZN}$.
(x) We observe that the third median also passes through the point of intersection G of first two medians.
(xi) Hence the three medians of the ΔXYZ pass through the same point G. That is, they are concurrent at G.

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Find the H.C.F. of the following expressions. ; EX #6.1 Q.1

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution: $39x^7y^3z = 3 \times 13\,x^7y^3z$; $91x^5y^6z^7 = 13 \times 7\,x^5y^6z^7$

H.C.F. $= 13\,x^5y^3z$

(ii) $102\,xy^2z$, $85\,x^2yz$ and $187\,xyz^2$

Solution: $102\,xy^2z = 2 \times 3 \times 17\,xy^2z$

$85\,x^2yz = 5 \times 17\,x^2yz$; $187\,xyz^2 = 11 \times 17\,xyz^2$

H.C.F. $= 17xyz$

Q2. Find the H.C.F. of the following expressions by factorization. ; EX #6.1 Q.2;(i , ii, iii)

(i) $x^2+5x+6,\ x^2-4x-12$

Solution: $x^2+5x+6 = x^2+3x+2x+6$

$= x(x+3)+2(x+3) \qquad = (x+3)(x+2)$

$x^2-4x-12 = x^2-6x+2x-12 = x(x-6)+2(x-6) \qquad = (x-6)(x+2)$

H.C.F. $= x+2$

(ii) $x^3-27,\ x^2+6x-27,\ 2x^2-18$

Solution: $x^3-27 = (x)^3-(3)^3 = (x-3)(x^2+3x+9)$

$x^2+6x-27 = x^2+9x-3x-27 = x(x+9)-3(x+9) \qquad = (x+9)(x-3)$

$2x^2-18 = 2(x^2-9) = 2(x^2-3^2) \quad = 2(x+3)(x-3)$

H.C.F. $= x-3$

(iii) $x^3-2x^2+x,\ x^2+2x-3,\ x^2+3x-4$

Solution: $x^3-2x^2+x = x(x^2-2x+1) = x(x-1)^2 = x(x-1)(x-1)$

$x^2+2x-3 = x^2+3x-x-3 = x(x+3)-1(x+3) = (x+3)(x-1)$

$x^2+3x-4 = x^2+4x-x-4 = x(x+4)-1(x+4) = (x+4)(x-1)$

H.C.F. $= x-1$

Q3. Find the H.C.F. of the following by division method. ; EX #6.1 Q.3;(iii)

(iii) $2x^5-4x^4-6x,\ x^5+x^4-3x^3-3x^2$

Solution: $2x^5-4x^4-6x = 2x(x^4-2x^3-3)$

$x^5+x^4-3x^3-3x^2 = x^2(x^3+x^2-3x-3)$

In this case H.C.F. of $2x$ and x^2 is x

Now we find H.C.F. of x^4-2x^3 and x^3+x^2-3x-3

$$\begin{array}{r|l} & x-3 \\ \hline x^3+x^2-3x-3 & x^4-2x^3-x^2+x-3 \\ & \underline{\pm x^4 \pm x^3 \mp 3x^3 \mp 3x} \\ & -3x^3+3x^2+3x-3 \\ & \underline{\mp 3x^3 \mp 3x^2 \pm 3x \pm 3} \end{array}$$

$$\begin{array}{r|l} & x+2 \\ \hline x^2-x-2 & x^3+x^2-3x-3 \\ & \underline{\pm x^3 \mp x^2 \mp 2x} \\ & 2x^2-x-3 \\ & \underline{\pm 2x^2 \mp 2x \mp 4} \\ & x+1 \end{array}$$

Then

$$\begin{array}{r|l} & x-2 \\ \hline x+1 & x^2-x-2 \\ & \underline{\pm x^2 \pm x} \\ & -2x-2 \\ & \underline{\mp 2x \mp 2} \\ & 0 \end{array}$$

$\text{H.C.F.} = x+1$

Hence the H.C.F. of the given expression is $x \times (x+1) = x^2 + x$

Q4. Find the L.C.M. of the following expressions. ; EX #6.1 Q.4

(i) $39x^7y^3z$ and $91x^5y^6z^7$

Solution: $39x^7y^3z = 3 \times 13\, x^7y^3z$; $91x^5y^6z^7 = 13 \times 7\, x^5y^6z^7$

$\text{L.C.M.} = 3 \times 7 \times 13\, x^7y^6z^7 = 273\, x^7y^6z^7$

(ii) $102\, xy^2z$, $85\, x^2yz$ and $187\, xyz^2$

Solution: $102\, xy^2z = 2 \times 3 \times 17\, xy^2z$; $85\, x^2yz = 5 \times 17\, x^2yz$

$187\, xyz^2 = 11 \times 17\, xyz^2$

$\text{L.C.M.} = 2 \times 3 \times 5 \times 11 \times 17x^2y^2z^2 = 5610\, x^2y^2z^2$

Q5. Find the L.C.M. of the following expressions by factorization. ; EX #6.1 Q.5;(ii, iii)

(ii) x^2+4x+4, x^2-4, $2x^2+x-6$

Solution: $x^2+4x+4 = (x+2)^2$

$x^2-4 = (x+2)(x-2)$

$2x^2+x-6 = 2x^2+4x-3x-6 = 2x(x+2)-3(x+2) = (x+2)(2x-3)$

$\text{L.C.M.} = (x+2)^2(x-2)(2x-3)$

(iii) $2(x^4-y^4), 3(x^3+2x^2y-xy^2-2y^3)$

Solution: $2(x^4-y^4) = 2(x^2-y^2)(x^2+y^2) = 2(x-y)(x+y) = 2(x^2-y^2)(x^2+y^2)(x^2+y^2)$

$3(x^3+2x^2y-xy^2-2y^3) = 3[x^2(x+2y)-y^2(x+2y)]$

$= 3(x+2y)(x^2+y^2) = 3(x+y)(x-y)(x+2y)$

$\text{L.C.M.} = 2 . 3(x-y)(x+y)(x^2+y^2)(x+2y)$

$= 6(x^2-y^2)(x^2+y^2)(x+2y) = 6(x^4-y^4)(x+2y)$

Q1. Use factorization to find the square root of the following expressions. ; EX #6.3 Q.1;(i , iv, vi , vii)

(i) $4x^2-12xy+9y^2$

Solution: $(2x)^2-2(2x)(3y)+(3y)^2 = (2x-3y)^2 = \sqrt{(2x-3y)^2}$

$\therefore$ Required square root is $\pm(2x-3y)$

(iv) $4(a+b)^2-12(a^2-b^2)+9(a-b)^2$

Solution: $= [2(a+b)]^2 - 2[2(a+b)][3(a-b)] + [3(a-b)]^2$

$= [2(a+b)-3(a-b)]^2 = (2a+2b-3a+3b)^2 = (5b-a)^2$

$\therefore$ Required square root is $\pm(5b-a)$

(vi) $\left(x+\frac{1}{x}\right)^2 - 4\left(x-\frac{1}{x}\right)$; $(x \neq 0)$

$= \left(x-\frac{1}{x}\right)^2 - 2\left(x-\frac{1}{x}\right).2 + (2)^2 \qquad = \left[\left(x-\frac{1}{x}\right)-2\right]^2$

$\therefore$ Required square root is $\pm\left[\left(x-\frac{1}{x}\right)-2\right]$

(vii) $\left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x+\frac{1}{x}\right)^2 + 12 \qquad (x \neq 0)$

Solution: $= x^4 + 2 + \frac{1}{x^4} - 4\left(x^2+2+\frac{1}{x^2}\right) + 12 = \left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x^2+\frac{1}{x^2}\right) - 8 + 12$

$= \left(x^2+\frac{1}{x^2}\right)^2 - 4\left(x^2+\frac{1}{x^2}\right) + 4 \qquad = \left(x^2+\frac{1}{x^2}\right)^2 - 2\left(x^2+\frac{1}{x^2}\right).2 + (2)^2$

$= \left[\left(x^2+\frac{1}{x^3}\right)-2\right]^2$ $\therefore$ Required square root is $\pm\left[\left(x^2+\frac{1}{x^2}\right)-2\right]$ **EX #6.3 Q.2;(i , iv, v)**

Q2. Use division method to find the square root of the following expressions.

(i) $4x^2 + 12xy + 9y^2 + 16x + 24y + 16$

Solution:

$$
\begin{array}{r|l}
 & 2x+3y+4 \\
\hline
2x & 4x^2+12xy+9y^2+16x+24y+16 \\
 & \pm 4x^2 \\
\hline
4x+3y & 12xy+16x+9y^2+24y+16 \\
 & \pm 12xy \quad \pm 9y^2 \\
\hline
4x+6y+4 & 16x \quad +24y+16 \\
 & \pm 16x \quad \pm 24y \pm 16 \\
\hline
 & 0
\end{array}
$$

$\therefore$ The square root is $\pm(2x+3y+4)$

(iv) $4 + 25x^2 - 12x - 24x^3 + 16x^4$

Solution:

$$
\begin{array}{r|l}
 & 4x^2-3x+2 \\
\hline
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\
\hline
8x^2-3x & -24x^3+25x^2 \\
 & \mp 24x^3 \pm 9x^2 \\
\hline
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm 16x^2 \mp 12x \pm 4 \\
\hline
 & 0
\end{array}
$$

$\therefore$ The square root is $\pm(4x^2-3x+2)$.

(v) $\frac{x^2}{y^2} - 10\frac{x}{y} + 27 - 10\frac{x}{y} + \frac{x^2}{y^2} \quad ; \quad (x \neq 0)(y \neq 0)$

Solution:

$$
\begin{array}{r|l}
 & \frac{x}{y}-5+\frac{y}{x} \\
\hline
\frac{x}{y} & \frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{x}{y}+\frac{x^2}{y^2} \\
 & \pm\frac{x^2}{y^2} \\
\hline
2\frac{x}{y}-5 & -10\frac{x}{y}+27 \\
 & \mp 10\frac{x}{y} \pm 25 \\
\hline
2\frac{x}{y}-10+\frac{y}{x} & 2-10\frac{x}{y}+\frac{y^2}{x^2}
\end{array}
$$

So the square root is $\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$.

Q1. Choose the correct answer. Review EX #6 Q.1

(i) H.C.F. of p^3q-pq^3 and $p^5q^2-p^2q^5$ is......
(a) $pq(p^2-q^2)$ (b) $pq(p-q)$ (c) $p^2q^2(p-q)$ (d) $pq(p^3-q^3)$

(ii) H.C.F. of $5x^2y^2$ and $20x^3y^3$ is......
(a) $5x^2y^2$ (b) $20x^3y^3$ (c) $100x^5y^5$ (d) $5xy$

(iii) H.C.F. of $x-2$ and x^2+x-6 is.......
(a) x^2+x-6 (b) $x+3$ (c) $x-2$ (d) $x+6$

(iv) H.C.F. of a^3+b^3 and a^2-ab+b^2 is......
(a) $a+b$ (b) a^2-ab+b^2 (c) $(a-b)^2$ (d) a^2+b^2

(v) H.C.F. of x^2-5x+6 and x^2-x-6 is.......
(a) $x-3$ (b) $x+2$ (c) x^2-4 (d) $x-2$

(vi) H.C.F. of a^2-b^2 and a^3-b^3 is......
(a) $a-b$ (b) $a+b$ (c) a^2+ab+b^2 (d) a^2-ab+b^2

(vii) H.C.F. of x^2+3x+2, x^2+4x+3 and x^2+5x+4 is......
(a) $x+1$ (b) $(x+1)(x+2)$ (c) $x+3$ (d) $(x+4)(x+1)$

(viii) L.C.M. of $15x^2$, $45xy$ and $30xyz$ is ....
(a) $90xyz$ (b) $90x^2yz$ (c) $15xyz$ (d) $15x^2yz$

(ix) L.C.M. of a^2+b^2 and a^4-b^4 is ......
(a) a^2+b^2 (b) a^2-b^2 (c) a^4-b^4 (d) $a-b$

(x) The product of algebraic expressions is equal to the ...... of their H.C.F. and L.C.M.
(a) Sum (b) Difference (c) Product (d) Quotient

(xi) Simplify $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}=\cdots$
(a) $\frac{4a}{9a^2-b^2}$ (b) $\frac{4a-b}{9a^2-b^2}$ (c) $\frac{4a+b}{9a^2-b^2}$ (d) $\frac{b}{9a^2-b^2}$

(xii) Simplify $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=\cdots$
(a) $\frac{a+7}{a-6}$ (b) $\frac{a+7}{a-2}$ (c) $\frac{a+3}{a-6}$ (d) $\frac{a-2}{a+3}$

(xiii) Simplify $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}=\cdots$
(a) $\frac{1}{a+b}$ (b) $\frac{1}{a-b}$ (c) $\frac{a-b}{a^2+b^2}$ (d) $\frac{a+b}{a^2+b^2}$

(xiv) Simplify $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=\cdots$
(a) $\frac{x}{x+y}$ (b) $\frac{y}{x+y}$ (c) $\frac{y}{x}$ (d) $\frac{x}{y}$

(xv) The square root of a^2-2a+1 is......
(a) $\pm(a+1)$ (b) $\pm(a-1)$ (c) $a-1$ (d) $a+1$

(xvi) What should be added to complete the square of x^4+64
(a) $8x^2$ (b) $-8x^2$ (c) $16x^2$ (d) $4x^2$

(xvii) The square root of $x^4+\frac{1}{x^4}+2$ is......
(a) $\pm\left(x+\frac{1}{x}\right)$ (b) $\left(x^2+\frac{1}{x^2}\right)$ (c) $\pm\left(x-\frac{1}{x}\right)$ (d) $\pm\left(x^2-\frac{1}{x^2}\right)$

Answers:

(i) b	(ii) a	(iii) c	(iv) b	(v) a
(vi) a	(vii) a	(viii) b	(ix) c	(x) c

GUESS PAPER & MODEL PAPER # 07
BASED ON UNIT # 7 (Reduced Syllabus)
LINEAR EQUATIONS AND INEQUALITIES

Unit 7	Linear Equations and Inequalities
Exercise 7.1	Q1(i, iii, v, vii, ix); Q2(i, ii, v, viii)
Exercise 7.2	Q1; Q2(ii, iv, v, vii)
Exercise 7.3	Q1(i, ii, iv, vii); Q2(i, ii, iii, viii)
Review Ex 7	Q1; Q2

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) Which of the following is the solution of the inequality $-4x \leq 11$?......
(A) -8 (B) -2 (C) -4 (D) None of these

(ii) A statement involving any of the symbols $<$, $>$, $\leq$ or $\geq$ is called ......
(A) equation (B) identity (C) inequality (D) linear equation

(iii) $x =$ ...... is a solution of the inequality $-2 < x < \frac{3}{2}$
(A) -5 (B) 3 (C) 0 (D) $\frac{3}{2}$

(iv) If x is no larger than 10, then......
(A) $x > 8$ (B) $x < 10$ (C) $x < 10$ (D) $x > 10$

(v) If the capacity c of an elevator is at most 1600 pounds, then......
(A) $c < 1600$ (B) $c > 1600$ (C) $c < 1600$ (D) $c > 1600$

(vi) $x = 0$ is a solution of the inequality......
(A) $x > 0$ (B) $3x + 5 < 0$ (C) $x + 2 < 0$ (D) $x - 2 < 0$

(vii) Solution set of $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$ is............
(A) $\left\{-\frac{1}{5}\right\}$ (B) $\left\{\frac{1}{5}\right\}$ (C) $\left\{\frac{13}{[illegible]}\right\}$ (D) $\left\{-\frac{4}{[illegible]}\right\}$

(ix) Solution set of $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$ is............
(A) {3} (B) {10} (C) {6} (D) {5}

(x) Solution set of $\frac{1}{2}|x+3|+21 = 9$ is............
(A) {0} (B) {−1} (C) {5} (D) { } or ϕ

(xi) Solution set of $3x+1 < 5x-4$ is............
(A) $\left\{x \mid x > \frac{3}{2}\right\}$ (B) $\left\{x \mid x > \frac{5}{2}\right\}$ (C) $\left\{x \mid x > \frac{2}{2}\right\}$ (D) $\left\{x \mid x > \frac{1}{2}\right\}$

(xii) Solution set of $4 - \frac{1}{2}x \geq -7 + \frac{1}{4}x$ is............
(A) $\left\{x \mid x > \frac{41}{2}\right\}$ (B) $\left\{x \mid x > \frac{46}{2}\right\}$ (C) $\left\{x \middle| x \leq \frac{44}{3}\right\}$ (D) $\left\{x \middle| x \leq \frac{42}{3}\right\}$

(xiii) Solution set of $|3x+14| - 2 = 5x$ is............
(A) {6, −2} (B) {−12, 0} (C) {2, 0} (D) {3, −1}

(xiv) Solution set of $-3 < \frac{1-2x}{5} < 1$ is............
(A) $\{x \mid x < 13\}$ (B) $\{x \mid 2 > x > -1\}$ (C) $\{x \mid 8 > x > -2\}$ (D) $\{x \mid x \geq 12\}$

(xv) Solution set of $\left|\frac{x+5}{2-x}\right| = 6$ is............
(A) $\left\{1, \frac{17}{5}\right\}$ (B) $\left\{-1, \frac{17}{5}\right\}$ (C) $\left\{1, \frac{12}{5}\right\}$ (D) $\left\{2, \frac{17}{2}\right\}$

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet–B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. (9 × 4 = 36)

(i) Solve the following equations. $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$; EX #7.1 Q.1;(i)

(ii) Solve for x. $\frac{1}{2}|3x+2| - 4 = 11$; EX #7.2 Q.2;(ii)

(iii) Solve the following inequalities. $-4 < 3x+5 < 8$; EX #7.3 Q.2;(i)

(iv) Solve for x. $\left|\frac{3x-5}{4}\right| - \frac{1}{3} = \frac{2}{3}$; EX #7.2 Q.2;(vii)

(v) Solve the following equations. $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$, $x \neq \pm 1$; EX #7.1 Q.1;(ix)

(vi) Solve the following equations. $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$; $x \neq -\frac{5}{2}$; EX #7.1 Q.1;(vii)

(vii) Solve for x. $|3+2x| = |6x-7|$; EX #7.2 Q.2;(iv)

(viii) Solve the following inequalities. 3x-2<2x+1<4x+17 ; EX #7.3 Q.2;(viii)

(ix) Solve the following inequalities. $-6 < \frac{x-2}{4} < 6$; EX #7.3 Q.2;(iii)

(x) Solve the following inequalities. $-5 \leq \frac{4-3x}{2} < 1$; EX #7.3 Q.2;(ii)

(xi) Solve for x. $|x+2| - 3 = 5 - |x+2|$; EX #7.2 Q.2;(v)

(xii) Solve the following equations. $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$; EX #7.1 Q.1;(v)

(xiii) Solve the following equations. $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$; EX #7.1 Q.1;(iii)

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Show that the points $M(-1,4), N(-5,3), P(1,-3)$ and $Q(5,-2)$ are the vertices of a parallelogram. ; EX #9.2 ; Q.9

Q.4 In an isosceles Δ, the base $\overline{BC} = 28\ cm$, and $\overline{AB} = \overline{AC} = 50\ cm$. If $\overline{AD} \perp \overline{BC}$, then find
(i) length of $\overline{AD}$ (ii) Area of ΔABC ; EX #15; Q.4

Q.5 In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides. ; Theorem # 15.1.1

Q.6 In a parallelogram ABCD, $m\overline{AB} = 10$ cm. The altitudes corresponding to sides AB and AD are respectively 7 cm and 8 cm. Find $\overline{AD}$. ; EX #16.1; Q.2

Q.7 Construct a right-angled isosceles triangle whose hypotenuse is 5.2 cm long.
EX #17.1 Q.4;(i)

SOLUTION OF GUESS PAPER & MODEL PAPER # 7
(Reduced Syllabus)

SECTION- A (MCQs)

i. B	ii. C	iii. C	iv. B	v. C	vi. D
vii. A	viii. C	ix. B	x. D	xi. B	xii. C
xiii. A	xiv. C	xv. A			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Solve the following equations. $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$; EX #7.1 Q.1;(i)

Solution: $\frac{2}{3}x - \frac{1}{2}x = x + \frac{1}{6}$

Multiplying both sides by 6 we get; $4x - 3x = 6x + 1 \Rightarrow x = 6x + 1$

$\Rightarrow$ $-5x = -\frac{1}{5}$; $\therefore$ Solution set = $\left\{-\frac{1}{5}\right\}$

(ii) Solve for x. $\frac{1}{2}|3x + 2| - 4 = 11$; EX #7.2 Q.2;(ii)

Solution: $\frac{1}{2}|3x+2| - 4 = 11$

$\frac{1}{2}|3x+2| = 15 \Rightarrow |3x+3| = 30$

The equation is equivalent to

$3x + 2 = 30$ or $3x + 2$

$x = 30 - 2$ or $3x = -30 - 2$

$3x = 28$ or $3x = -32$

$x = \frac{28}{3}$ or $x = \frac{-32}{3}$

$\therefore$ Solution set = $\left\{\frac{28}{3}, -\frac{32}{3}\right\}$

(iii) Solve the following inequalities. $-4 < 3x + 5 < 8$; EX #7.3 Q.2;(i)

Solution: The given equality represents two inequalities

$3x > -9 \Rightarrow \quad x > -\frac{9}{3} \Rightarrow \quad x > -3 \Rightarrow \quad -3 < x$ .... (i)

The second inequality $3x + 5 < 8$ gives $\quad 3x < 8 - 5$

$3x < 3 \Rightarrow$ Or $\quad x < 1$ .... (ii)

Combining (i) and (ii), we have $\quad -3 < x < 1$

$\therefore$ Solution set is $\{x \mid -3 < x < 1\}$

(iv) Solve for x. $\left|\frac{3x-5}{4}\right| - \frac{1}{3} = \frac{2}{3}$; EX #7.2 Q.2;(vii)

Solution: $\left|\frac{3x-5}{4}\right| - \frac{1}{3} = \frac{2}{3}$

$\left|\frac{3-5x}{4}\right| = \frac{2}{3} + \frac{1}{8} = 1$

The given equation is equivalent to $\left|\frac{3-5x}{4}\right| = \pm 1$ or $3 - 5x = \pm 4$

i.e. $3 - 5x = 4$ or $3 - 5x = -4$

$-5x = 4$ or $-5x = -4 - 3$

$-5x = 1$ or $-5x = -7$

$x = -\frac{1}{5}$ or $x = \frac{7}{5}$

$\therefore$ Solution set $= \left\{-\frac{1}{7}, \frac{7}{5}\right\}$

(v) Solve the following equations. $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$, $x \neq \pm 1$; EX #7.1 Q.1;(ix)

Solution: $\frac{2}{x^2-1} - \frac{1}{x+1} = \frac{1}{x+1}$

Multiplying both sides by $x^2 - 1$

$2 - (x - 1) = x - 1 \Rightarrow \quad 2 - x + 1 = x - 1 \Rightarrow \quad -x - x = -1 - 2 - 1$

$-2x = -4 \Rightarrow \quad x = 2$; So solution set $= \{2\}$

(vi) Solve the following equations. $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{4x+10}$; $x \neq -\frac{5}{2}$; EX #7.1 Q.1;(vii)

Solution: $\frac{2x}{2x+5} = \frac{2}{3} - \frac{5}{2(2x+5)}$

Multiplying both sides by $6(2x + 5)$ we get ; $\quad 6(2x) = 4(2x + 5) - 15$

Or $\quad 12x = 8x + 20 - 15$

$x = \frac{5}{4}$; So solution set $= \left\{\frac{5}{4}\right\}$

(vii) Solve for x. $|3 + 2x| = |6x - 7|$; EX #7.2 Q.2;(iv)

Solution: $|3 + 2x| = |6x - 7|$

The given equation is equivalent to $\quad 3 + 2x = \pm(6x - 7)$

i.e. $3 + 2x = 6x - 7$ or $3 + 2x = -6(6x - 7)$

i.e. $2x - 6x = -7$ or $2x + 6x = 7 - 3$

i.e. $-4x = -10$ or $8x = 4$

$x = \frac{5}{2}$ or $x = \frac{1}{2}$; Solution set $\left\{\frac{5}{2}, \frac{1}{2}\right\}$

(viii) Solve the following inequalities. $3x - 2 < 2x + 1 < 4x + 17$; EX #7.3 Q.2;(viii)

Solution: $3x - 2 < 2x + 1 < 4x + 17$

This is equivalent to $\quad 3x - 2 < 2x + 1$ And $\quad 2x + 1 < 4x + 17$

The first inequality gives $\quad 3x - 2 < 2x + 1$

or $\quad 3x - 2x < 1 + 2 \Rightarrow$ or $\quad x < 3$............ (i)

The second inequality gives $\quad 2x + 1 < 4x + 17$

$\therefore$ solution set is $\{x| -8 < x < 3\}$

(ix) Solve the following inequalities. $-6 < \frac{x-2}{4} < 6$; EX #7.3 Q.2;(iii)

Solution: $-6 < \frac{x-2}{4} < 6$

This inequality is equivalent to two inequalities $-6 < \frac{x-2}{4}$

And $\frac{x-2}{4} < 6$

The first inequality gives $-24 < x - 2$

$\Rightarrow$ $-24 + 2 < x \Rightarrow$ $-22 < x$ .... (i)

The second inequality gives $x - 2 < 24$

$x < 24 + 2$ Or $x < 26$ ....(ii)

Combining (i) and (ii) we have $-22 < x < 26$

$\therefore$ Solution set is $\{x \mid -22 < x < 26\}$

(x) Solve the following inequalities. $-5 \le \frac{4-3x}{2} < 1$; EX #7.3 Q.2;(ii)

Solution: $-5 \le \frac{4-3x}{2} < 1$

The given inequality represents two inequalities $-5 \le \frac{4-3x}{2}$ And $\frac{4-3x}{2} < 1$

The first inequality gives $-5 \le \frac{4-3x}{2}$

$-10 \le 4 - 3x \Rightarrow -10 - 4 \le -3x \Rightarrow 14 \ge 3x \Rightarrow \frac{14}{3} \ge x$ .... (i)

The second inequality gives $\frac{4-3x}{2} < 1$

$\Rightarrow$ $4 - 3x < 2 \Rightarrow 4 - 2 < 3x \Rightarrow 2 < 3x \Rightarrow \frac{2}{3} < x$ .... (ii)

Combining (i) and (ii) $\frac{2}{3} < x \le \frac{14}{3}$; $\therefore$ Solution set is $\left\{x \mid \frac{2}{3} < x < \frac{14}{3}\right\}$

(xi) Solve for x. $|x+2| - 3 = 5 - |x+2|$; EX #7.2 Q.2;(v)

Solution: $|x+2| - 3 = 5 - |x+2|$

$\Rightarrow$ $|x+2| + |x+2| = 5 + 3 \Rightarrow 2|x+2| = 8 \Rightarrow |x+2| = 4$

This equation is equivalent to $x + 2 = 4$ *or* $x + 2 = -4$

$x = 2$ *or* $x = -6$; $\therefore$ Solution set = $\{2, -6\}$

(xii) Solve the following equations. $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$; EX #7.1 Q.1;(v)

Solution: $\frac{5(x-3)}{6} - x = 1 - \frac{x}{9}$

Multiplying both sides by 18 we get ; $15(x-3) - 18x = 18 - 2x$

Or $15x - 45 - 18x = 18 - 2x \Rightarrow$ $15x - 18x + 2x = 18 + 45$

$-x = 63 \Rightarrow$ $x = -63$; $\therefore$ Solution set = $\{-63\}$

(xiii) Solve the following equations. $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$; EX #7.1 Q.1;(iii)

Solution: $\frac{1}{2}\left(x - \frac{1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1}{2} - 3x\right)$

$\frac{1}{2}\left(\frac{6x-1}{6}\right) + \frac{2}{3} = \frac{5}{6} + \frac{1}{3}\left(\frac{1-6x}{2}\right)$ Or $\frac{6x-1}{12} + \frac{2}{3} = \frac{5}{6} + \frac{1-6x}{6}$

Multiplying both sides by 12

(xiv) Solve each equation and check for extraneous solution, If any. $\sqrt{\frac{x+1}{2x+5}} = 2$; $x \neq \frac{5}{2}$

EX #7.1 Q.2;(viii)

Solution: $\sqrt{\frac{x+1}{2x+5}} = 2$; Squaring both sides $\frac{x+1}{2x+5} = 4$

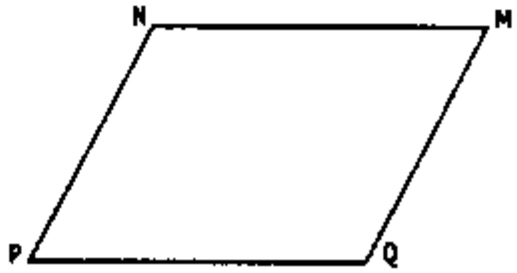
$x + 1 = 4(2x + 5) \Rightarrow x - 1 = 8x + 20 \Rightarrow x - 8x = 20 - 1$

$-7x = 19 \Rightarrow x = -\frac{19}{7}$; $\therefore$ Solution set $= \left\{-\frac{19}{7}\right\}$

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Show that the points $M(-1,4), N(-5,3), P(1,-3)$ and $Q(5,-2)$ are the vertices of a parallelogram. ; EX #9.2 ; Q.9



Solution:

Points are $M(-1,4), N(-5,3), P(1,-3)$ and $Q(5,-2)$

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|MN| = \sqrt{(-1+5)^2 + (4-3)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17}$

$|PQ| = \sqrt{(5-1)^2 + (4-3)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17}$

$|NP| = \sqrt{(1+5)^2 + (-3-3)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$|MQ| = \sqrt{(5+1)^2 + (-2-4)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36+36} = 6\sqrt{2}$

$|QN| = \sqrt{(5+5)^2 + (-2-3)^2} = \sqrt{(10)^2 + (-5)^2} = \sqrt{100+25} = \sqrt{125} = 5\sqrt{5}$

$|NP|^2 + |PQ|^2 = 72 + 17 = 89 \neq 125 = |QN|^2$

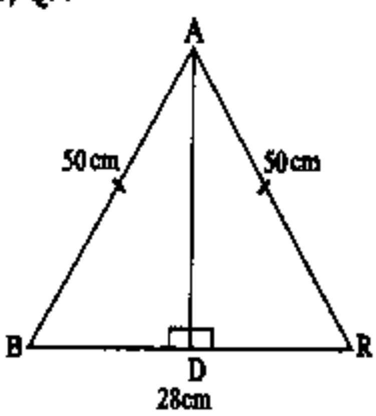
But $|MN| = |PQ| = |NQ| = |MQ|$

Hence the given points from a parallelogram.

Q.4 In an isosceles Δ, the base $\overline{BC} = 28\ cm$, and $\overline{AB} = \overline{AC} = 50\ cm$. If $\overline{AD} \perp \overline{BC}$, then find

(i) length of $\overline{AD}$ (ii) Area of ΔABC ; EX #15; Q.4

Solution:



(i) $\overline{AD} \perp \overline{BC}$

$\therefore$ D is mid point for $\overline{BC}$

So $m\overline{BD} = \frac{1}{2}(28) = 14\ cm$

From right angled ΔABD

$(m\overline{AB})^2 = (m\overline{BD})^2 + (m\overline{AD})^2$

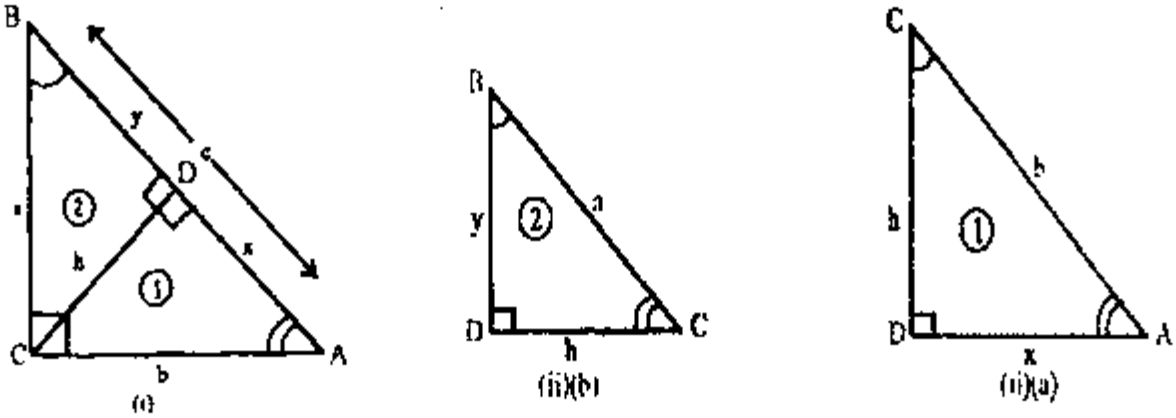
$(50)^2 = (14)^2 + (m\overline{AD})^2$

$(m\overline{AD})^2 = (50)^2 - (14)^2 = 2500 - 196 = 2304$

$m\overline{AD} = \sqrt{2304} = 48cm$

(ii) Area of $\Delta ABC = \frac{1}{2}$ base $\times$ altitude $= \frac{1}{2}(m\overline{BC}) \times (m\overline{AD})$

Q.5 In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides. ; Theorem # 15.1.1



Given:

ΔACB is a right angle triangle in which $m\angle C = 90^\circ$ and $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$

To Prove: $c^2 = a^2 + b^2$

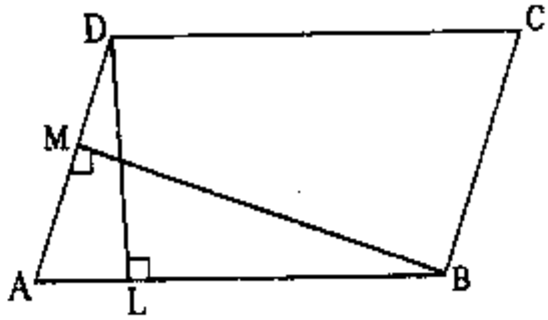
Construction:

Draw $\overline{CD}$ perpendicular from C on $\overline{AB}$. Let $m\overline{CD} = h$, $m\overline{AD} = x$ and $m\overline{BD} = y$. Line segment $\overline{CD}$ splits ΔABC into two triangles ADC and BDC which are separately shown in figure ii (a) and ii (b) respectively.

Proof:

Statements	Reasons
In the correspondence	
$\Delta ADC \leftrightarrow \Delta ACB$	
$\angle A \cong \angle A$	Refer to figure ii (a) and (i) common-self congruent
$\angle ADC \cong \angle ACB$	Construction given both measure 90°
$\angle C \cong \angle B$	$\angle C$ and $\angle B$, complements of $\angle A$
$\therefore$ $\Delta ADC \cong \Delta ACB$	Congruency of three angles
$\therefore$ $\frac{x}{b} = \frac{b}{c}$	Measure of corresponding sides of similar triangles is similar.
Again in the correspondence	Refer to figure ii(b) and (i)
$\Delta BDC \leftrightarrow \Delta BCA$	
$\angle B \cong \angle B$	Common self congruent
$\angle BDC \cong \angle BCA$	Construction given, both measure 90°
$\angle C \cong \angle A$	$\angle C$ and $\angle A$, complements of $\angle B$
$\therefore$ $\Delta BDC \cong \Delta BCA$	Congruency of three angles.
$\therefore$ $\frac{y}{a} = \frac{a}{c}$	Sides of similar triangles are proportional. (Theorem 6)
or $^{c} y = \frac{a}{c^2}$ ..... (ii)	
But $y + x = c$	Supposition
$\frac{a^2}{c} + \frac{b^2}{c} = c$	By (i) and (ii)

Q.6 In a parallelogram ABCD, $m\overline{AB} = 10$ cm. The altitudes corresponding to sides AB and AD are respectively 7 cm and 8 cm. Find $\overline{AD}$. ; EX #16.1; Q.2



Solution:

Given:

ABCD is a parallelogram.

$m\overline{AB} = 10$ cm, $\overline{DL}$ and $\overline{BM}$ are altitudes

$m\overline{DL} = 7$ cm, $m\overline{BM} = 8$ cm

To prove: $m\overline{AD} = ?$

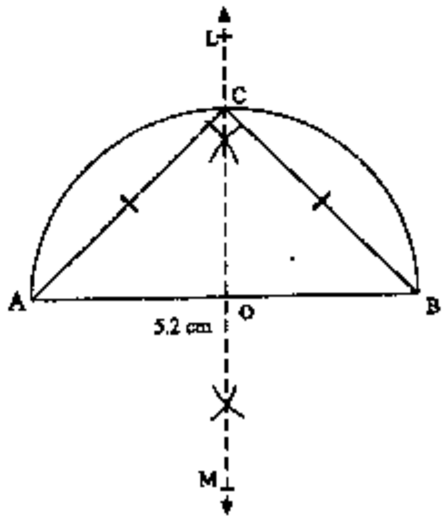
Proof: Area of a parallelogram = base ×altitude

Area of a parallelogram ABCD $m\overline{AB} \times m\overline{DL} = m\overline{AD} \times m\overline{BM}$

$\therefore \quad 10 \times 7 = m\overline{AD} \times 8$

$\therefore \quad m\overline{AD} = \frac{10 \times 7}{8} = \frac{35}{8} = 8.75 \text{ cm}$

Q.7 Construct a right-angled isosceles triangle whose hypotenuse is 5.2 cm long.
EX #17.1 Q.4;(i)



Solution:

Construction:

(i) Draw a line segment $m\overline{AB} = 5$ cm.

(ii) Draw $\overleftrightarrow{LM}$ the right bisector of $\overline{AB}$ cutting it at the point O.

(iii) With centre at the point O and $\overline{AB}$ as diameter draw a semi-circle to cut $\overleftrightarrow{LM}$ at the point C.

(iv) Join C to A and B.
So the required triangle is ΔABC.

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q2. Solve each equation and check for extraneous solution, if any. ; EX #7.1 Q.2;(i , ii, v)

Extraneous solution: When raising each side of the equation to a certain power may produce a nonequivalent equation that has more solutions than the original equation. These additional solutions are called extraneous solutions. We must check our answer(s) for such solutions when working with radical equations.

(i) $\sqrt{3x+4} = 2$ **Solution:** Taking square of both sides

$3x + 4 = 4 \Rightarrow \quad 3x = 4 - 4 \Rightarrow \quad 3x = 0$ Or $x = 0$; $\therefore$ Solution set = $\{0\}$

(ii) $\sqrt[3]{2x-4} - 2 = 0$

Solution: $\therefore \sqrt[3]{3x-4} = 2$; Taking cube of both sides; $2x - 4 = 2^3 = 8$

Or $2x = 8 + 4 \Rightarrow \quad 2x = 12$ Or $x = 6$; $\therefore$ Solution set = $\{6\}$

(v) $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Solution: Taking cube of both sides $2x + 3 = x - 2$

$2x - x = -2 - 3 \Rightarrow \quad x = -5$; $\therefore$ Solution set = $\{-5\}$

Q1. Identify the following statements as True or False ; EX #7.2 Q.1

(iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$.
(iv) The equation $|x - 4| = -4$ has no solution.
(v) The equation $|2x - 3| = 5$ is equivalent to $2x - 3 = 5$ or $2x + 3 = 5$.

Answers:

(i) T	(ii) F	(iii) T	(iv) T	(v) F

Q1. **Solve the following inequalities. ; EX #7.3 Q.1;(i , ii ,iv , vii)**

(i) $3x + 1 < 5x - 4$

Solution: $3x - 5x < -4 - 1 \Rightarrow -2x < -5 \Rightarrow -2x > 5 \Rightarrow x > \frac{5}{2}$; ∴ Solution set is $\left\{x \mid x > \frac{5}{2}\right\}$

(ii) $4x - 10.3 \leq 21x - 1.8$

Solution: $4x - 21x \leq -1.8 + 10.3$

$17x \leq 8.5 \Rightarrow \quad x \geq -0.5$; ∴Solution set is $\{x \mid x \geq -0.5\}$

(iv) $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

Solution: $x - 0 + 4x \geq 10 - \frac{7}{2} \Rightarrow 5x - 6x \geq 10 - \frac{7}{2}$

$-x \geq \frac{13}{2} \Rightarrow -x \geq 6.5 \Rightarrow x \leq -6.5$; ∴ Solution set is $\{x|x \leq -6.5\}$

(vii) $3(x - 1) - (x - 2) > -2(x + 4)$

Solution: $3x - 3 - x + 2 > -2x - 8 \Rightarrow \quad 2x - 1 > -2x - 8 \Rightarrow \quad 2x + 2x < -8 + 1$

$4x > -\frac{7}{4}$; ∴ Solution set is $\left\{x \mid x > -\frac{7}{4}\right\}$

Q1. **Choose the correct answer. ; Review EX #7 Q.1**

(i) **Which of the following is the solution of the inequality $-4x \leq 11$?......**
(a) -8 (b) -2 (c) -4 (d) None of these

(ii) **A statement involving any of the symbols $<$, $>$, $\leq$ or $\geq$ is called ......**
(a) equation (b) identity (c) inequality (d) linear equation

(iii) **x = ... is a solution of the inequality $-2 < x < \frac{3}{2}$**
(a) -5 (b) 3 (c) 0 (d) $\frac{3}{2}$

(iv) **If x is no larger than 10, then......**
(a) $x > 8$ (b) $x < 10$ (c) $x < 10$ (d) $x > 10$

(v) **If the capacity c of an elevator is at most 1600 pounds, then......**
(a) $c < 1600$ (b) $c > 1600$ (c) $c < 1600$ (d) $c > 1600$

(vi) **$x = 0$ is a solution of the inequality......**
(a) $x > 0$ (b) $3x + 5 < 0$ (c) $x + 2 < 0$ (d) $x - 2 < 0$

Answers:

(i) b	(ii) c	(iii) c	(iv) b	(v) c	(vi) d

Q2. **Identify the following statements as True or False. ; Review EX #7 Q.2**

(i) The equation $3x - 5 = 7 - x$ is a linear equation.
(ii) The equation $x - 0.3x = 0.7x$ is an identity.
(iii) The equation $-2x + 3 = 8$ is equivalent to $-2x = 11$.
(iv) To eliminate fractions, we multiply each side of an equation by the L.C.M. of denominators.
(v) $4(x + 3) = x + 3$ is a conditional equation.
(vi) The equation $2(3x + 5) = 6x + 12$ is ah inconsistent equation.
(vii) To solve $\frac{2}{3}x = 1.2$, we should multiply each side by $\frac{2}{3}$.
(viii) Equations having exactly the same solution are called equivalent equations.

GUESS PAPER & MODEL PAPER # 08
BASED ON UNIT # 8 (Reduced Syllabus)
LINEAR GRAPHS AND THEIR APPLICATION

Unit 8	Linear Graphs and their Applications
Exercise 8.1	Q1; Q2(i, ii, iii, iv, v, vi, vii, xiii, xiv); Q5
Exercise 8.2	Q3(a, b, c); Q4
Exercise 8.3	Q1; Q2; Q3
Review Ex 8	Q1; Q2

NOTE:

- All Class work will be given for revision as H.W.
- The MCQ's Portion of the annual paper will be taken from MCQ's exercise at the end of the chapters: so MCQ's will be done in class by class teacher.

SECTION-A

Time allowed: 20 Minutes **Marks: 15**

Note: Section-A is compulsory. All parts of this section are to be answered on the question paper itself. It should be completed in the first 20 minutes and handed over to the Centre Superintendent. Deleting/overwriting is not allowed. Do not use lead pencil.

Q.1 Encircle the correct option i.e. A / B / C / D. All parts carry equal marks.

(i) If $(x-1, y+1) = (0, 0)$, then (x, y) is
(A) $(1, -1)$ (B) $(-1, 1)$ (C) $(1, 1)$ (D) $(-1, -1)$

(ii) If $(x, 0) = (0, y)$, then (x, y) is
(A) $(0, 1)$ (B) $(1,0)$ (C) $(0, 0)$ (D) $(1, 1)$

(iii) Point $(2, -3)$ lies in quadrant
(A) I (B) II (C) III (D) IV

(iv) Point $(-3, -3)$ lies in quadrant
(A) I (B) II (C) III (D) IV

(v) If $y = 2x + 1$, $x = 2$ then y is
(A) 2 (B) 3 (C) 4 (D) 5

(vi) Which ordered pair satisfy the equation $y = 2x$.
(A) $(1, 2)$ (B) $(2, 1)$ (C) $(2, 2)$ (D) $(0, 1)$

(vii) Point $(5, -2)$ lies in quadrant
(A) I (B) II (C) III (D) IV

(viii) Mid-point of the points $(2, -2)$ and $(-2, 2)$ is
(A) $(2, 2)$ (B) $(-2, -2)$ (C) $(0, 0)$ (D) $(1, 1)$

(ix) A triangle having all sides different is called
(A) Isosceles (B) Scalene (C) Equilateral (D) None of these

(x) A triangle having two sides congruent is called......

(xii) Distance between points (0, 0) and (1, 1) is
(A) 0 (B) 1 (C) 2 (D) $\sqrt{2}$

(xiii) The medians of a triangle cut each other in the ratio......
(A) 4 : 1 (B) 3 : 1 (C) 2:1 (D) 1:1

(xiv) The right bisectors of the three sides of a triangle are......
(A) congruent (B) collinear (C) concurrent (D) parallel

(xv) The diagonals of a parallelogram......each other.
(A) bisect (B) trisect (C) bisect at right angle (D) none of these

Time allowed: 2:40 hours **Total Marks: 60**

Note: Attempt any nine parts from Section 'B' and any three questions from Section 'C' on the separately provided answer book. Use supplementary answer sheet i.e. Sheet--B if required. Write your answers neatly and legibly. Log book and graph paper will be provided on demand.

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. $(9 \times 4 = 36)$

(i) Draw the graph of the following. $x = 2$; EX #8.1 Q.2;(i)
(ii) Draw the graph of the following. $x = -3$; EX #8.1 Q.2;(ii)
(iii) Draw the graph of the following. $y = -1$; EX #8.1 Q.2;(iii)
(iv) Draw the graph of the following. $y = 3$; EX #8.1 Q.2;(iv)
(v) Draw the graph of the following. $y = 0$; EX #8.1 Q.2;(v)
(vi) Draw the graph of the following. $x = 0$; EX #8.1 Q.2;(vi)
(vii) Draw the graph of the following. $y = 3x$; EX #8.1 Q.2;(vii)
(viii) Draw the graph of the following. $x - 3y + 1 = 0$; EX #8.1 Q.2;(xiii)
(ix) Draw the graph of the following. $3x - 2y + 1 = 0$; EX #8.1 Q.2;(xiv)
(x) Sketch the graph for following line. $x - 3y + 2 = 0$; EX #8.2 Q.3;(a)
(xi) Sketch the graph for following line. $3x - 2y - 1 = 0$; EX #8.2 Q.3;(b)
(xii) Sketch the graph for following line. $2y - x + 2 = 0$; EX #8.2 Q.3;(c)
(xiii) Solve the following pair of equations in x and y graphically.
$x + y = 0$ and $2x - y + 3 = 0$; EX #8.3 ; Q.1
(xiv) Solve the following pair of equations in x and y graphically.
$x - y + 1 = 0$ and $x - 2y = -1$; EX #8.3 ; Q.2

SECTION – C (Marks 24)

Note: Attempt any THREE questions. Each question carries equal marks. $(3 \times 8 = 24)$

Q.3 Construct the following $\Delta's$ XYZ. Draw their three medians and show that they are concurrent? $m\overline{YZ} = 4.1$ cm, $m\angle X = 75°$, $m\angle Y = 60°$; EX #17.2 Q.4;(i)

Q.4 Construct the following triangles ABC. Draw the perpendicular bisectors of their sides and verify their concurrency. Do you meet inside the triangle?
$m\overline{AB} = 5.3$ cm, $m\angle A = 45°$, $m\angle B = 30°$; EX #17.2 Q.3;(i)

Q.5 Construct the following $\Delta's$ PQR. Draw their altitudes and show that they are concurrent.
$m\overline{PQ} = 6$ cm, $m\overline{QR} = 4.5$ cm, $m\overline{PR} = 5.5$; EX #17.2 Q.2;(i)

Q.6 Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area. ; ; Theorem # 16.1.1

Q.7 The end point of a line segment PQ is $(-3, 6)$ and its mid-point is $(5, 8)$. Find the coordinates of the end point Q. ; EX #9.3 ; Q.2

SOLUTION OF GUESS PAPER & MODEL PAPER # 8
(Reduced Syllabus)

SECTION- A (MCQs)

i. A	ii. C	iii. D	iv. C	v. D	vi. A
vii. D	viii. C	ix. B	x. D	xi. C	xii. D
xiii. C	xiv. C	xv. A			

SECTION – B (Marks 36)

Q.2 Attempt any NINE parts from the following. All parts carry equal marks. (9 × 4 = 36)

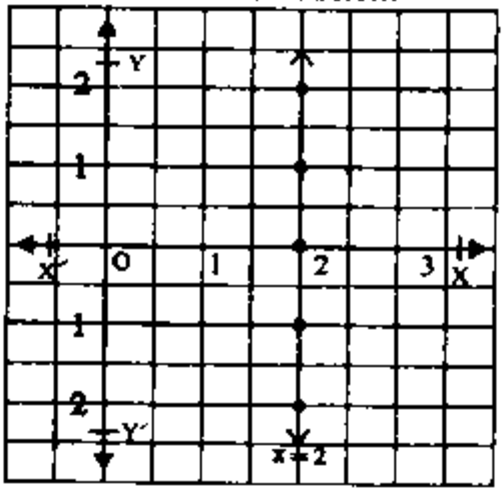
(i) Draw the graph of the following. $x = 2$; EX #8.1 Q.2;(i)

Solution: $x = 2$

Table for the points of the equation x = 2 is as under:

x	2	2	2	2	2	2	2
y	...	−2	−1	0	1	2	...

Thus the graph of the equation x = 2 is as shown below.



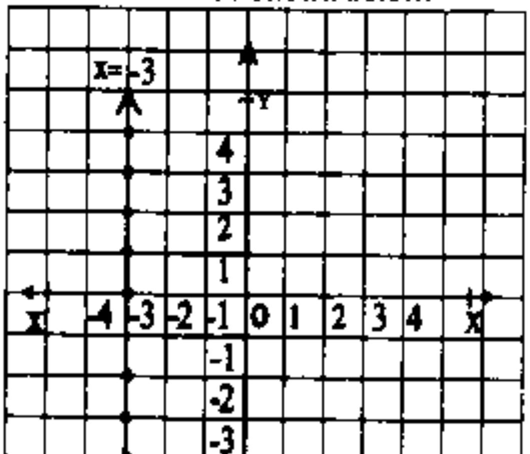
(ii) Draw the graph of the following. $x = -3$; EX #8.1 Q.2;(ii)

Solution: $x = -3$

Table for the points of the equation x = −3 is as under:

x	-3	-3	-3	-3	-3	-3	-3
y	...	−2	−1	0	1	2	...

Thus the graph of the equation x = −3 is as shown below.



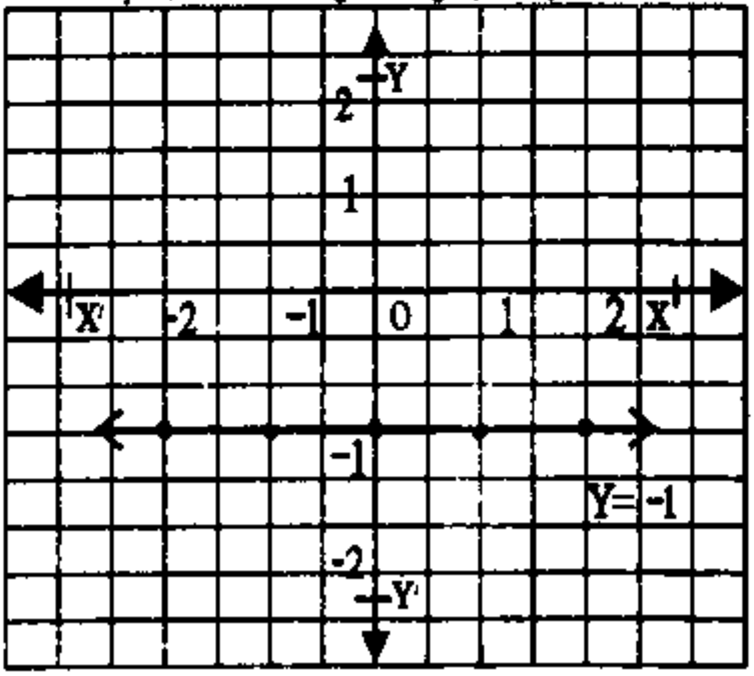
(iii) **Draw the graph of the following.** $y = -1$ **; EX #8.1 Q.2;(iii)**

Solution: $y = -1$

Table for the points of the equation y = –1 is as under:

x	...	–2	–1	0	1	2	...
y	–1	–1	–1	–1	–1	–1	–1

Plotting these points and joined them we get the graph of y = –1 as under:



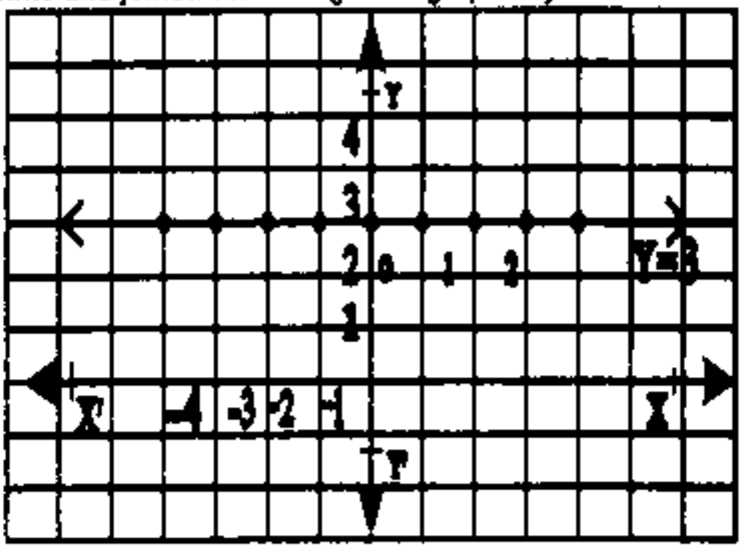
(iv) **Draw the graph of the following.** $y = 3$ **; EX #8.1 Q.2;(iv)**

Solution: $y = 3$

Table for the points of the equation y = 3 is as under:

x	...	–2	–1	0	1	2	...
y	3	3	3	3	3	3	3

Plotting these points and joined them we get the graph of y = 3 as under:

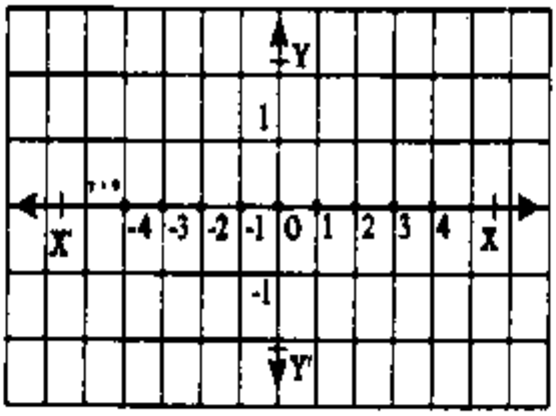


(v) **Draw the graph of the following.** $y = 0$ **; EX #8.1 Q.2;(v)**

Solution: $y = 0$

Table for the points of the equation y = 0 is as under:

x	...	–2	–1	0	1	2	...
y	0	0	0	0	0	0	0



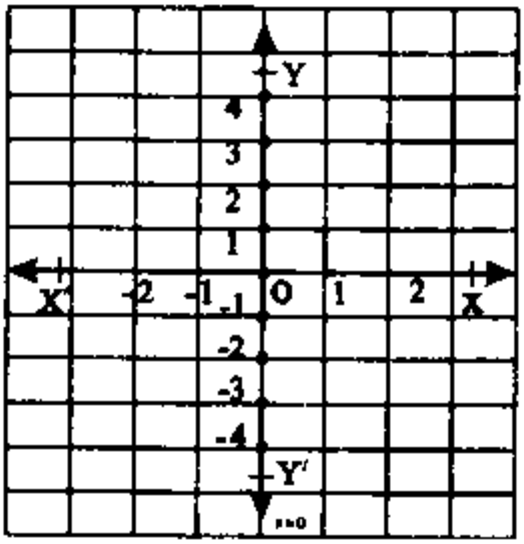
(vi) Draw the graph of the following. $x = 0$; EX #8.1 Q.2;(vi)

Solution: $x = 0$

Table for the points of the equation x = 0 is as under:

x	0	0	0	0	0	0	0
y	...	−2	−1	0	1	2	...

Plotting these points we see that all the points are on y-axis. So the graph of the equation x = 0 is y-axis as shown below.



(vii) Draw the graph of the following. $y = 3x$; EX #8.1 Q.2;(vii)

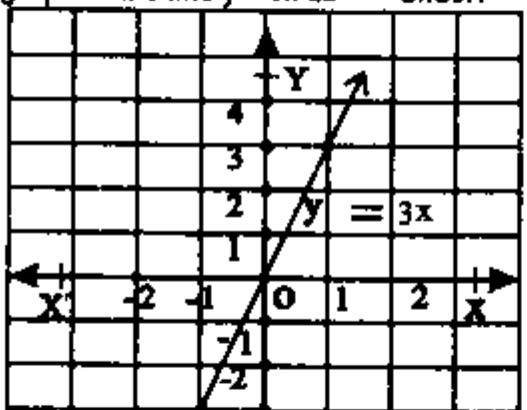
Solution: $y = 3x$

Table for the points of the equation y = 3x is as under:

x	− 2	− 1	0	1	2
y	− 6	−3	0	3	6

The points (x, y) are plotted in the plane as shown below:

Joining them we get the graph of the line y = 3x as under:



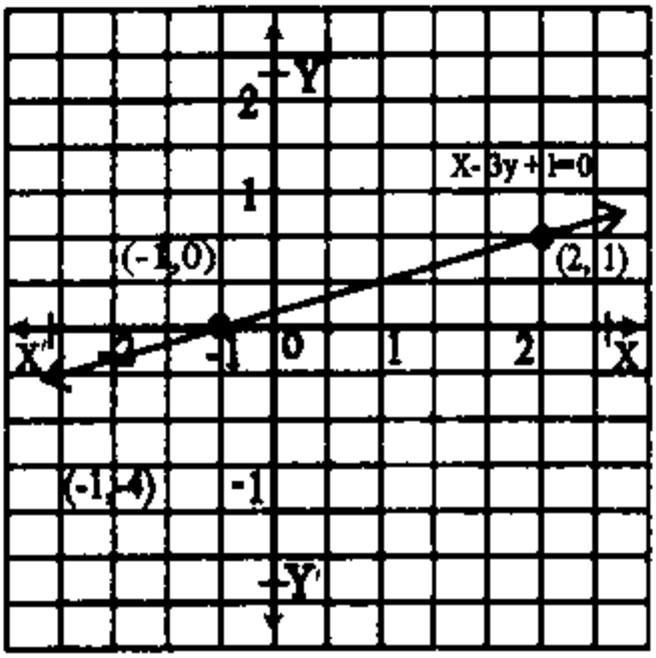
(viii) Draw the graph of the following. $x - 3y + 1 = 0$; EX #8.1 Q.2;(xiii)

Solution: $x - 3y + 1 = 0$ i.e. $x = 3y - 1$ or $y = \frac{x+1}{3}$

Table for the points of equation is as under:

x	-2	-1	0	1	2
y	-4	-2	0	2	4

The points are plotted in the plane. By joining the plotted points we get the graph of the equation as under:



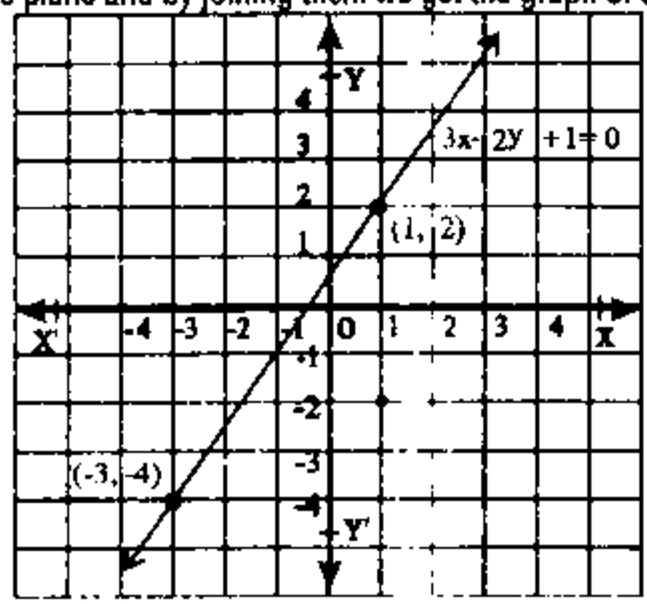
(ix) Draw the graph of the following. $3x - 2y + 1 = 0$; EX #8.1 Q.2;(xiv)

Solution: $3x - 2y + 1 = 0$ or $y = \frac{3x+1}{2}$

Table for the points of the equation is as under:

x	-3	-2	-1	0	1
y	-4	$-2\frac{1}{2}$	-1	5	2

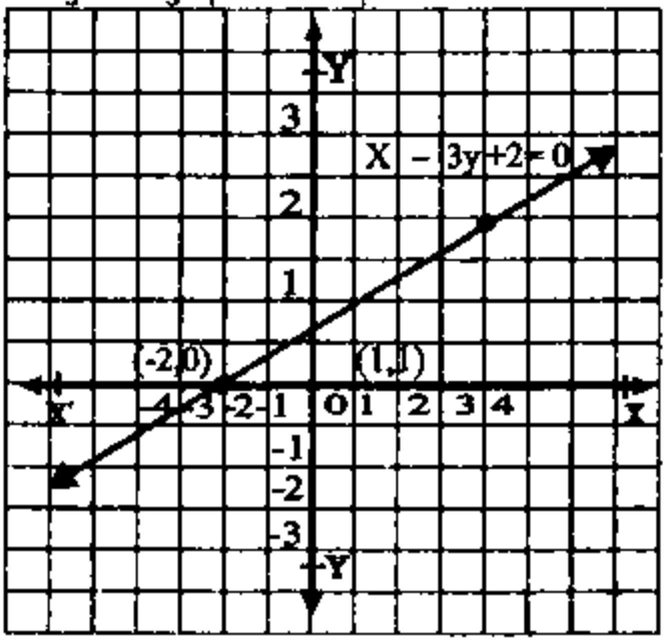
The points are plotted in the plane and by joining them we get the graph of the equation as under:



(x) Sketch the graph for following line ... ; EX #8.2 Q.2;(a)

x	-2	-1	0	1	2	3	4
y	0	0.3	0.66	1	1.3	2.66	2

Plotting these points we get the graph of the equation as under:



(xi) **Sketch the graph for following line.** $3x - 2y - 1 = 0$; EX #8.2 Q.3;(b)

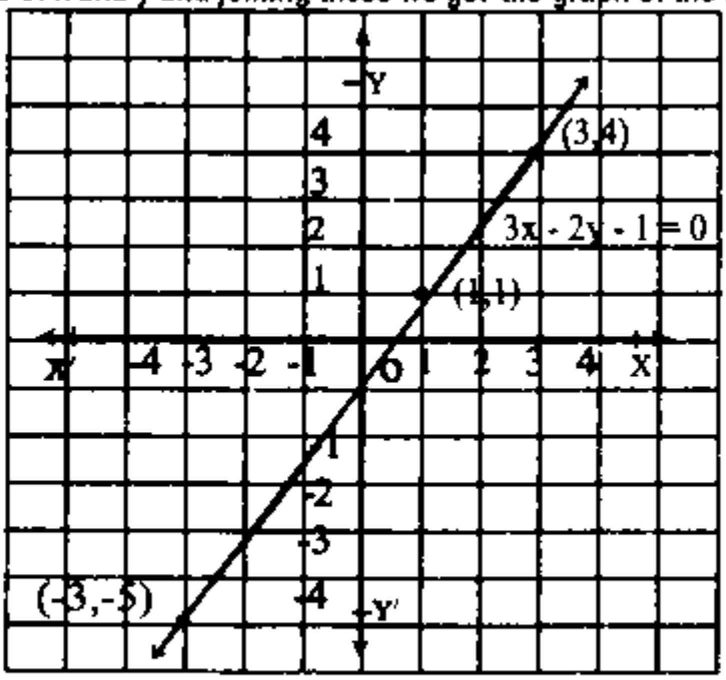
Solution: $3x - 2y - 1 = 0$

or $3x - 1 = 2y$ or $y = \frac{3x-1}{2}$

We tabulate the values of (x, y) as under:

x	-3	-2	-1	0	1	2	3
y	-5	-3.5	-2	-0.5	1	2.5	4

Plotting the values of x and y and joining these we get the graph of the equation as under:



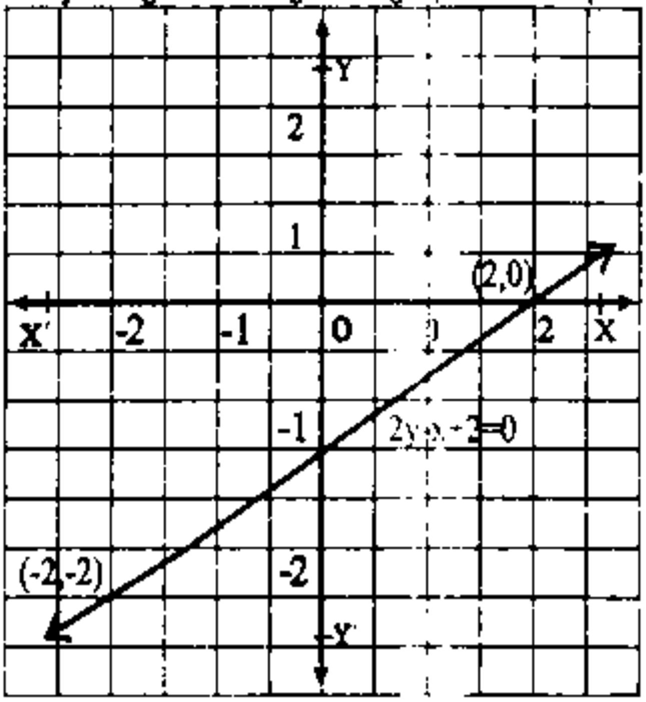
(xii) **Sketch the graph for following line.** $2y - x + 2 = 0$; EX #8.2 Q.3;(c)

Solution: $2y - x + 2 = 0$

or $2y = x - 2$ or $y = \frac{x-2}{2}$

We calculate the values (x, y) as under:

Plotting these points and joining them we get the graph of the equation as under:



(xiii) **Solve the following pair of equations in x and y graphically.**
$x + y = 0$ and $2x - y + 3 = 0$; EX #8.3 ; Q.1

Solution: Let the system of the equations be

$x + y = 0$ (i)

$2x - y + 3 = 0$ (ii)

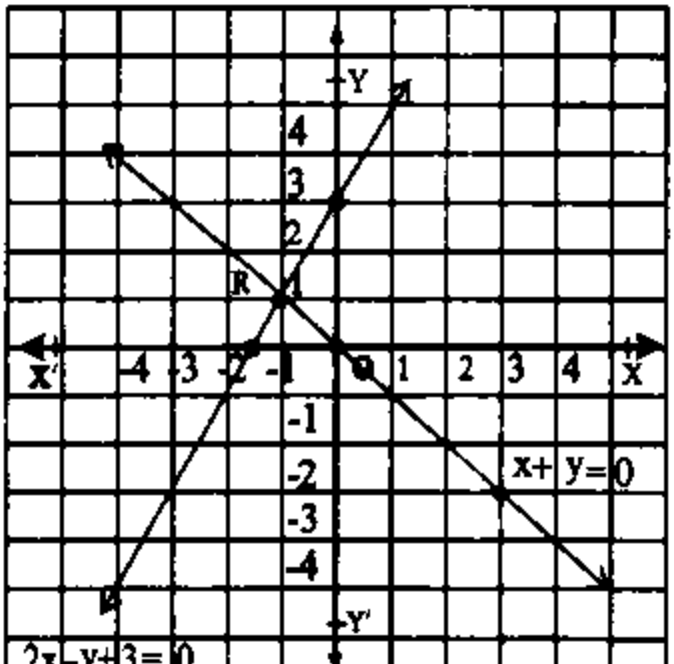
For (i) $y = -x$ the table of values is

x	0	-1	2
y	0	1	-2

For (ii) $y = 2x + 3$, the table of values is

x	0	-1.5	-1
y	3	0	1

By plotting the points we get the following graph.



(xiv) Solve the following pair of equations in x and y graphically.
$x - y + 1 = 0$ and $x - 2y = -1$; EX #8.3 ; Q.2

Solution: Let the system of equations be

$x - y + 1 = 0$ ....(i) ; $x - 2y = -1$ ....(ii)

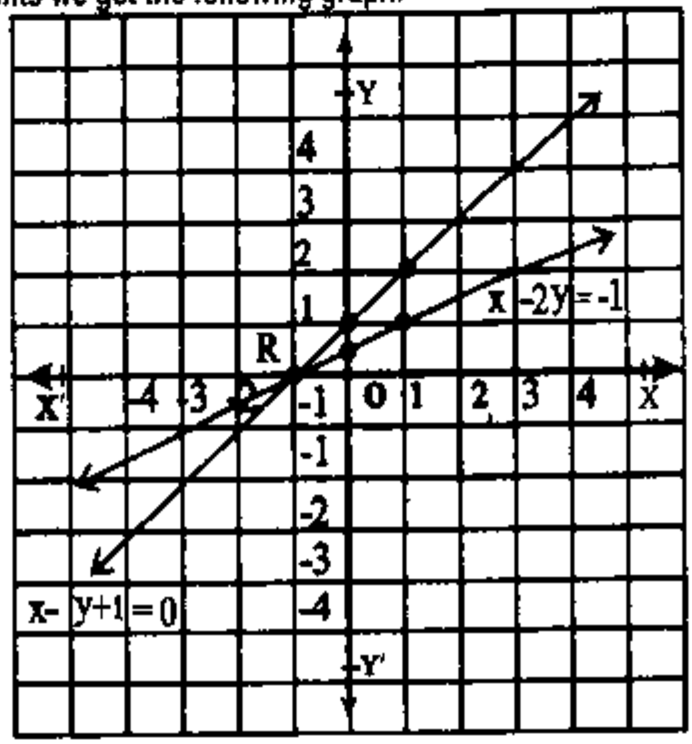
For (i) $y = x + 1$, the table of values is

x	0	-1	1
y	1	0	2

For (ii) $y = \frac{x+1}{2}$, the table of values is

x	0	-1	1
y	0.5	0	1

By plotting the points we get the following graph.



The solution of the system is the point R where the two lines meet i.e. R(–1, 0) x = –1, y = 0.

SECTION – C (Marks 24)

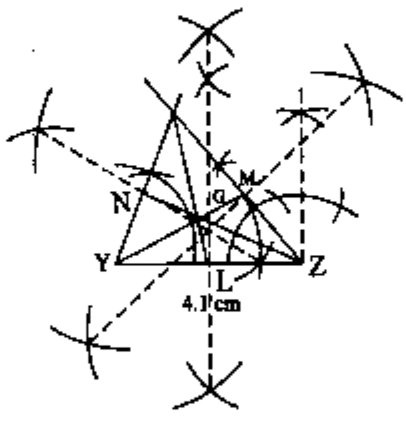
Note: Attempt any THREE questions. Each question carries equal marks. **(3 × 8 = 24)**

Q.3 **Construct the following Δ's XYZ. Draw their three medians and show that they are concurrent? $m\overline{YZ} = 4.1$ cm, $m\angle X = 75°$, $m\angle Y = 60°$; EX #17.2 Q.4;(i)**

Solution: $m\angle X = 75°$, $m\angle Y = 60°$

So $m\angle Z = 180° - 135° = 45°$

Construction:



(i) Take $m\overline{YZ} = 4.1$ cm.
(ii) At the point Y make $m\angle XYZ = 60°$.
(iii) At the point Z make $m\angle XYZ = 45°$.
(iv) The terminal sides of the two angles meet at X and we get the ΔXYZ.
(v) Draw perpendicular bisectors of the sides $\overline{XY}$, $\overline{YZ}$ and $\overline{XZ}$ of the ΔXYZ and mark their mid points L, M and N respectively.
(vi) Join X to L to get the median $\overline{XL}$.
(vii) Join Y to M to get the median $\overline{YM}$.

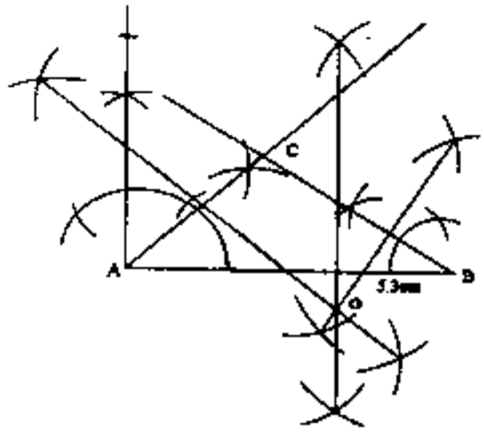
(x) We observe that the third median also passes through the point of intersection of first two medians.

(xi) Hence the three medians of the ΔXYZ pass through the same point G i.e. they are concurrent at the point G.

Q.4 Construct the following triangles ABC. Draw the perpendicular bisectors of their sides and verify their concurrency. Do you meet inside the triangle?
$m\overline{AB} = 5.3$ cm, $m\angle A = 45°$, $m\angle B = 30°$; EX #17.2 Q.3;(i)

Solution:

Construction:

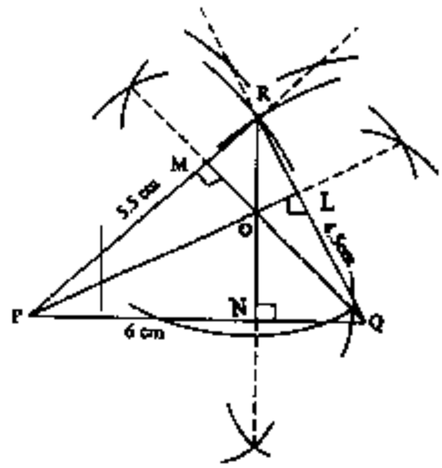


(i) Take $m\overline{AB} = 5.3$ cm.
(ii) At the point A make $m\angle BAC = 45°$.
(iii) At the point B make $m\angle ABC = 30°$.
(iv) The terminal sides of these angles meet at C and ABC is the required triangle.
(v) Draw perpendicular bisectors of $\overline{BC}$ and $\overline{CA}$ meeting each other at the point O.
(vi) Now draw the perpendicular bisector of third side $\overline{AB}$.
(vii) We observe that it also passes through O, the point of intersection of first two perpendicular bisectors.
(vii) Hence the three perpendicular bisectors of sides of ΔABC are concurrent at O.

Q.5 Construct the following $\Delta's$ PQR. Draw their altitudes and show that they are concurrent.
$m\overline{PQ} = 6$ cm, $m\overline{QR} = 4.5$ cm, $m\overline{PR} = 5.5$; EX #17.2 Q.2;(i)

Solution:

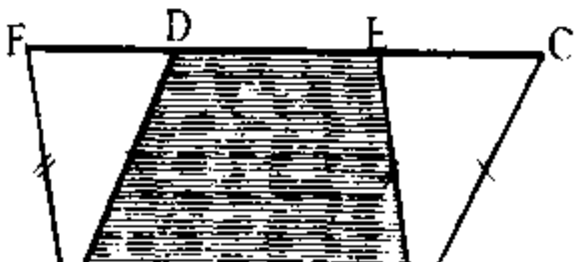
Construction:



(i) Take $m\overline{PQ} = 6$ cm.
(ii) With P as centre and radius equal to 5.5 cm draw an arc.
(iii) With Q as centre and radius equal to 4.5 cm draw another arc to cut the first arc at R.
(iv) Join $\overline{PR}$ and $\overline{QR}$ to complete the triangle ΔPQR.
(v) From the vertex P draw $\overline{PL} \perp \overline{QR}$.
(vi) From the vertex Q draw $\overline{QM} \perp \overline{PR}$. These two altitudes meet in the point O inside the ΔPQR.
(vii) Now from the third vertex 'R' draw $\overline{RN} \perp \overline{PQ}$.
(viii) We observe that this third altitude also passes through the point of intersection O of the first two altitudes.
(viii) Hence the three altitudes of the ΔPQR are concurrent at O.

Q.6 Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area. ; ; Theorem # 16.1.1

Solution:



Given: Two parallelograms ABCD and ABEF having the same base AB and between the same parallel lines AB and DE.

To Prove: Area of parallelogram ABCD = Area of parallelogram ABEF

Proof:

Statements	Reasons
area of (parallelogram ABCD)	
= area of (quadrilateral ABED) + area of (ΔCBE)..... (1)	Area addition axiom
= area of (quadrilateral ABED) + area of (ADAF) .... (2)	Area addition axiom
$m\overline{CB} = m\overline{DA}$	Opposite sides of a parallelogram
$m\overline{BE} = m\overline{AF}$	Opposite sides of a parallelogram
∠CBE = ∠DAF	Opposite sides of a parallelogram
∴ ΔCBE ≅ ΔDAF	S.A.S. congruent Axiom
area of (ΔCBE) = area of (ΔDAF) .... (3)	Congruent area axiom
Hence area of (parallelogram ABCD)	
= area of (parallelogram ABEF)	from (1), (2) and (3)

Q.7 The end point of a line segment PQ is $(-3,6)$ and its mid-point is $(5,8)$. Find the coordinates of the end point Q. ; EX #9.3 ; Q.2

Solution:

P(-3, 6) M (5, 8) Q (x, y)

Let Q be the point (x,y), $M(5,8)$ is the mid point of PQ

∴ by mind point formula we have $x = \frac{x_1+x_2}{2} \Rightarrow 5 = \frac{-3+x}{2}$

$10 = -3 + x \Rightarrow 10 + 3 = x \Rightarrow x = 13$

Now $y = \frac{y_1+y_2}{2} \Rightarrow 8 = \frac{6+y}{2} \Rightarrow 16 = 6 + y \Rightarrow 16 - 6 = y \Rightarrow y = 10$

Hence point Q is (13, 10)

IMPORTANT QUESTIONS & ANSWERS
(Reduced Syllabus)

Q1. Determine the quadrant of the coordinate plane in which the following points lie: $P(-4,3), Q(-5,-2), R(2,2)$ and $S(2,-6)$. ; EX #8.1 Q.1

Solution: $P(-4,3)$ lies in Second quadrant. ; $Q(-5,-2)$ lies in Third quadrant.
$R(2,2)$ lies in First quadrant. ; $S(2,-6)$ lies in Fourth quadrant.

Q5. Verify whether the following point lies on the line 2x − y + 1 = 0 or not.
(i) (2, 3) (ii) (0, 0) (iii) (−1, 1) (iv) (2, 5) (v) (5, 3)

(i) (2, 3) EX #8.1 Q.5

Solution: The line is 2x - y + 1 = 0 for the point (2, 3)
2(2) - 3 + 1 = 4 - 3 + 1 = 2 ≠ 0 ; ∴ point does not lie on the line

(ii) (0, 0)

Solution: The line is 2x - y + 1 ; For the point (0, 0) ; 2(0) - 0 + 1 = 0 - 0 = 1 ≠ 0
∴ point does not lie on the line

(iii) (−1, 1)

Solution: The line is 2x - y + 1 = 0 for (-1, 1)

$2(2) - 5 + 1 = 4 - 5 + 1 = 0$; $\therefore$ the point lies on the line

(v) (5, 3)

Solution: The line is $2x - y + 1 = 0$ for the point (5, 3)

$2(5) - 3 + 1 = 10 - 3 + 1 = 8 \neq 0$; $\therefore$ the point does not lie on the line.

Q4. Draw the graph for following relations. EX #8.2 Q.4

(i) One mile = 1.6 km (ii) One Acre = 0.4 Hectare

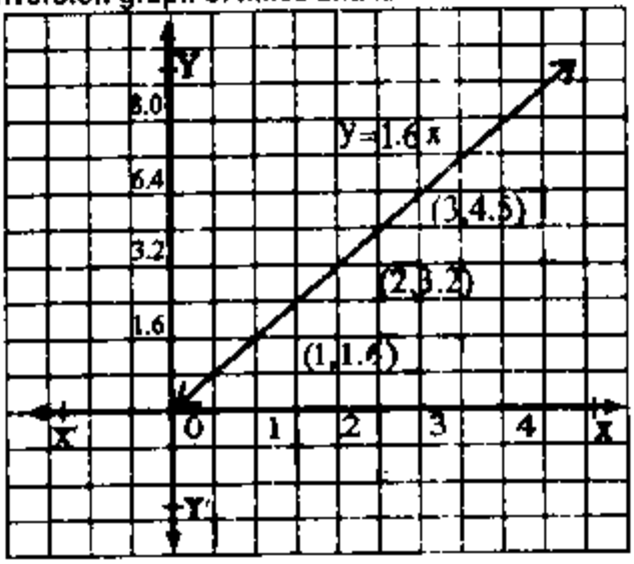
(iii) $F = \frac{9}{5}C + 32$ (iv) One Rupee = $\frac{1}{86}\$$

Solution: (i) One mile = 1.6 km

Let $y = 1.6x$; We tabulate value of x and y as under:

x	0	1	2	3	4
y	0	1.6	3.2	4.8	6.4

Mile is taken along x-axis and Km along y-axis. We plot the point (x, y) and joining them we get the graph of $y = 1.6x$ i.e. conversion graph of miles and km.



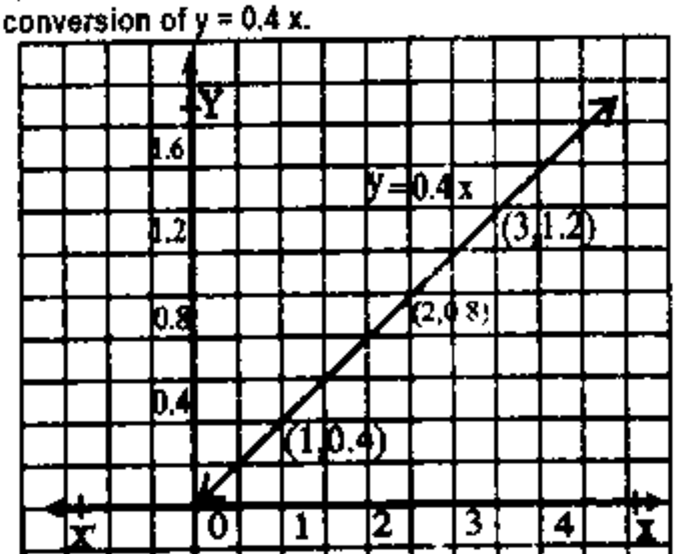
(ii) **On Acre = 0.4 Hectare**

If Acre is measured along x-axis and hectare along y-axis then $y = 0.4x$

The ordered pairs are tabulated in the following table.

x	0	1	2	4
y	0	0.4	0.8	1.2

The corresponding points (0, 0), (1, 1.4), (2, 0.8) etc, are plotted in the xy-plane. Join of which forms the graph of conversion of $y = 0.4x$.

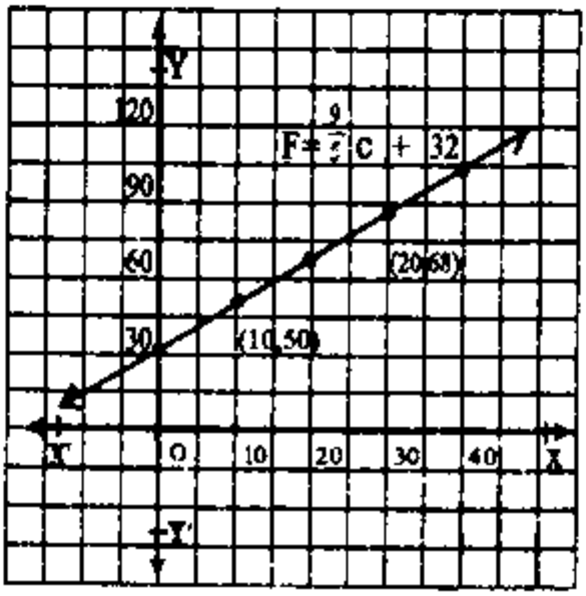


(iii) $F = \frac{9}{5}C + 32$

We tabulate the values of C and F

C	0°	10°	20°	30°	40°
F	32°	50°	68°	86°	104°

Plotting these points and joining them we get the required graph of $F = \frac{9}{5}C + 32$ as under:



(iv) One Rupee = $\frac{1}{86}$ \$

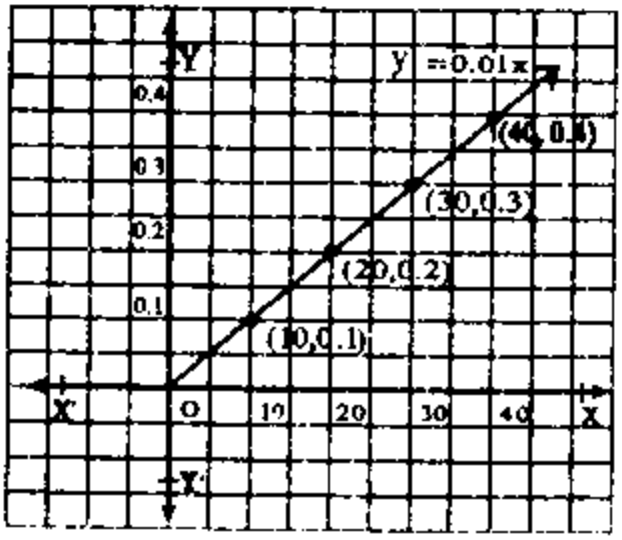
or One Rupee = 0.01\$

If \$ y is an expression of Rs. X, expressed under the rule y = 0.01 x

We tabulate the value of x and y as under:

x	0	10	20	30	40
y	0	0.1	0.2	0.3	0.4

Plotting the points corresponding to the ordered pairs (x, y) from the table and joining them we get the required graph.



EX #8.3 ; Q.3

Solve the following pair of equations in x and y graphically.

Q3. $2x + y = 0$ and $x + 2y = 2$

Solution: Let the system of equations be

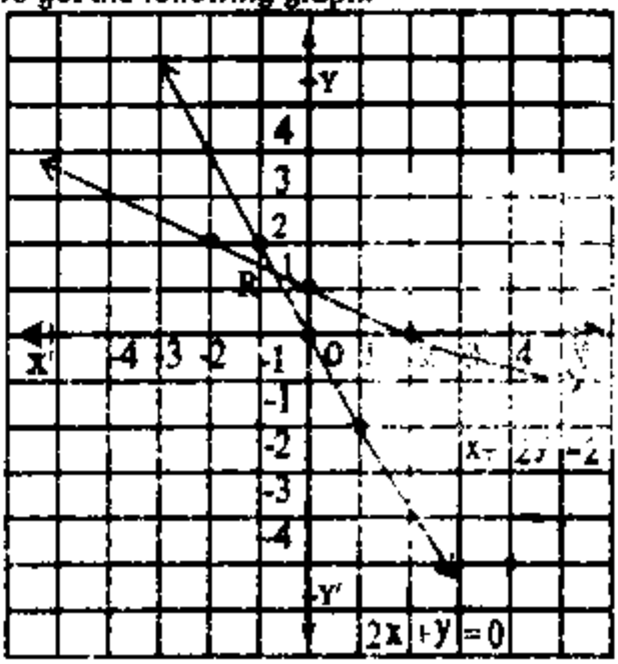
$2x + y = 0$ (i) $x + 2y = 2$ (ii)

For (i) $y = -2x$, the table of values is

For (ii) $y = \frac{2-x}{2}$, the table of values is

x	0	[illegible]	-2
y	1	0	2

By plotting the points we get the following graph.



The solution of the system is the point R where the two lines meet i.e. R $\left(-\frac{2}{3}, \frac{4}{3}\right)$

$x = -\frac{2}{3}$, $y = \frac{4}{3}$

Q1. Choose the correct answer. ; Review EX #8 ; Q.1

(i) If (x − 1, y + 1) = (0, 0), then (x, y) is
(a) (1, −1) (b) (−1, 1) (c) (1, 1) (d) (−1, −1)

(ii) If (x, 0) = (0, y), then (x, y) is
(a) (0, 1) (b) (1,0) (c) (0, 0) (d) (1, 1)

(iii) Point (2, −3) lies in quadrant
(a) I (b) II (c) III (d) IV

(iv) Point (−3, −3) lies in quadrant
(a) I (b) II (c) III (d) IV

(v) If y = 2x + 1, x = 2 then y is
(a) 2 (b) 3 (c) 4 (d) 5

(vi) Which ordered pair satisfy the equation y = 2x.
(a) (1, 2) (b) (2, 1) (c) (2 2) (d) (0 1)

Answers:

(i) a	(ii) c	(iii) d	(iv) c	(v) d	(vi) a

Q2. Identify the following statements as True or False. ; Review EX #8 ; Q.2

(i) The point O(0, 0) is in quadrant II.
(ii) The point P(2, 0) lies on x-axis.
(iii) The graph of x = −2 is a vertical line.
(iv) 3 - y = 0 is a horizontal line.
(v) The point Q(−1, 2) is in quadrant III.
(vi) The point R(−1, −2) is in quadrant IV.
(vii) y = x is a line on which origin lies.
(viii) The point P(1, 1) lies on the line x + y = 0
(ix) The point S(1, −3) lies in quadrant III.
(x) The point R(0, 1) lies on the x-axis.

Answers:

Important Questions & Answers (Reduced Syllabus Geometry Portion)

UNIT # 9,10,11,12,13,14,15,16,17

Q1. Find the distance between the following pairs of points. EX #9.1 ; Q.1;(a , d , e)

Solution: (a) $A(9,2),\ B(7,2)$

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(7-9)^2 + (2-2)^2} \quad = \sqrt{(-2)^2 + (0)^2} \quad = \sqrt{4+0} = \sqrt{4} = 2$

(d) $A(-4, \sqrt{2}),\ B(-4,-3)$

Solution: $AB = \sqrt{[-4-(-4)]^2 + [-3-\sqrt{2}]^2} \quad = \sqrt{(-4+4)^2 + (-3-\sqrt{2})^2}$

$= \sqrt{(0)^2 4(-3-\sqrt{2})^2} = |-3-\sqrt{2}| \quad = 3+\sqrt{2}$

(e) $A(3,11),\ B(3,4)$

Solution: $|AB| = \sqrt{(3-3)^2 + [-4-(-11)]^2} \quad = \sqrt{(0)^2 + (7)^2} = 7$

(f) $A(0,0),\ B(0,5)$

Solution: $|AB| = \sqrt{(0-0)^2 + (-5-0)^2} \quad = \sqrt{(0)^2 + (-5)^2} = \sqrt{0+25} \quad = \sqrt{25} = 5$

Q2. Let P be the point on x-axis with x-coordinate a and Q be the point on y-axis with y-coordinate b as given below. Find the distance between P and Q. EX #9.1 ; Q.2;(i , ii)

(i) $a = 9,\quad b = 7$

Solution: $\therefore\ P\ is\ (9,0) and\ Q\ is\ (0,7)$

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|PQ| = \sqrt{(0-9)^2 + (7-0)^2} \quad = \sqrt{81+49} = \sqrt{130}$

(ii) $a = 2,\quad b = 3$

Solution: $\therefore\ P\ is\ (2,0) and\ Q\ is\ (0,3)$

$Distance\ formula = d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(0-2)^2 + (3-0)^2} \quad = \sqrt{4+9} = \sqrt{13}$

Q1. Find the mid-point of the line segment joining each of the following pairs of points.

(a) A(9,2), B(7,2) EX #9.3 ; Q.1;(a , c , f)

Solution: Mid- point M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{9+7}{2}, \frac{2+2}{2}\right)$ Or $\left(\frac{16}{2}, \frac{4}{2}\right)$ Or $(8,\ 2)$

(c) A(3,-11),B (3,-4)

Solution: Mid- point M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$ Or $\left(\frac{-2}{2}, \frac{2}{2}\right)$ Or $(-1,1)$

(f) A(0,0), B(0,-5)

Solution: Midpoint M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$; $\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$ or $\left(\frac{0}{2}, \frac{-5}{2}\right) = (0,-2.5)$

Q1. Choose the correct answer. Review EX #9 ; Q.1

(i) Distance between points (0, 0) and (1, 1) is

(iii) **Mid-point of the points (2, 2) and (0, 0) is**
(a) (1, 1) (b) (1, 0) (c) (0, 1) (d) (-1, -1)

(iv) **Mid-point of the points (2, -2) and (-2. 2) is**
(a) (2, 2) (b) (-2, -2) (c) (0, 0) (d) (1, 1)

(v) **A triangle having all sides equal is called**
(a) Isosceles (b) Scalene (c) Equilateral (d) None of these

(vi) **A triangle having all sides different is called**
(a) Isosceles (b) Scalene (c) Equilateral (d) None of these

Answers:

(i) d	(ii) c	(iii) a	(iv) c	(v) c	(vi) b

Q2. Answer the following, which is true and which is false. ; Review EX #9 ; Q.2

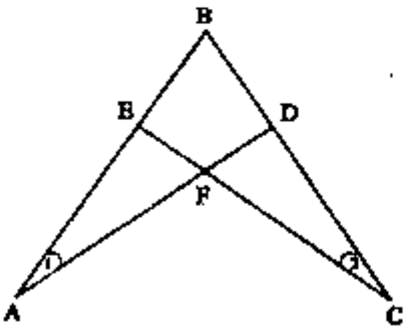
(i) A line has two end points.
(ii) A line segment has one end point.
(iii) A triangle is formed by three collinear points.
(iv) Each side of a triangle has two collinear vertices.
(v) The end points of each side of a rectangle are collinear.
(vi) All the points that lie on the x-axis are collinear.
(vii) Origin is the only point collinear with the points of both the axes separately.

Answers:

(i) F	(ii) F	(iii) F	(iv) T	(v) T
(vi) T	(vii) T			

EX #10.1 ; Q.1

Q1. **In the given figure,** $\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$
Prove that $\Delta ABD \cong \Delta CBE$



Solution:

Given:
In the given figure $\angle 1 \cong \angle 2$ and $\overline{AB} \cong \overline{CB}$

To prove: $\Delta ABD \cong \Delta CBE$

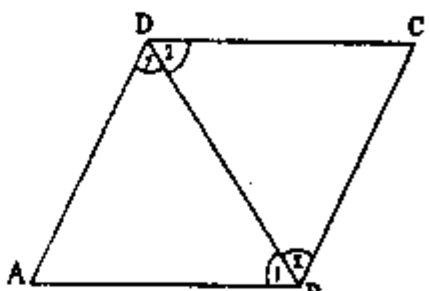
Proof:

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle BAD \cong \angle BCE$	Given $\angle 1 \cong \angle 2$
$\angle ABD \cong \angle CBE$	Common
$\therefore \quad \Delta ABD \cong \Delta CBE$	S.A.A $\cong$ S.A.A

EX #10.3 ; Q.1

Q1. **In the given figure,** $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}.$

Prove that $\angle A \cong \angle C, \angle ABC \cong \angle ADC$



Solution:

Given: In the figure $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

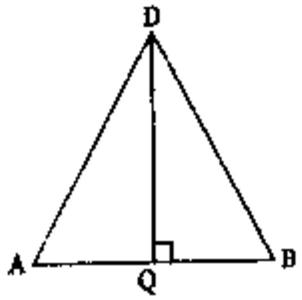
To prove: $\angle \cong \angle C$
$\angle ABC \cong \angle ADC$

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{CD}$	Given
$\overline{BD} \cong \overline{DB}$	Common
$\therefore$ $\Delta ABC \cong \Delta CDB$	S.S.S. ≅ S.S.S.
$\therefore$ $\angle A \cong \angle C$	Corresponding sides of ≅ Δs
$\left.\begin{matrix}\hat{1} \cong \hat{2} \\ \text{and} \\ \angle x \cong \angle y\end{matrix}\right\}$	Corresponding sides of ≅ Δs.
$\therefore$ by adding above equations	
$\hat{1}+\hat{x}=\hat{2}+\hat{y}$	Addition of angles
or $\angle ABC \cong \angle ADC$	

EX #10.4 ; Q.1

Q1. **In Δ PAB of figure, $\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$, Prove that $\overline{AQ} \cong \overline{BQ}$, and $\angle APQ \cong \angle BPQ$.**

Solution:



Given:

In ΔPAB,

$\overline{PQ} \perp \overline{AB}$, and $\overline{PA} \cong \overline{PB}$

To prove:

$\overline{AQ} \cong \overline{BQ}$

and $\angle APQ \cong \angle BPQ$

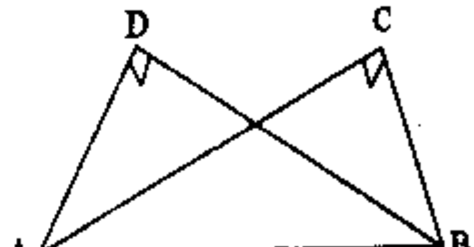
Proof:

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\angle AQP \cong \angle BQP$	Given $PQ \perp AB$
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore$ $\Delta APQ \cong \Delta BPQ$	H.S. ≅ H.S.
So $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of ≅ Δs.
and $\angle APQ \cong \angle BPQ$	Corresponding sides of ≅ Δs.

EX #10.4 ; Q.2

Q2. **In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$.**
Prove that $\overline{AC} \cong \overline{BD}$, and $\angle BAC \cong \angle ABD$.

Solution:



Given:

In the figure,

$m\angle C = m\angle D = 90^\circ$

and $\overline{BC} \cong \overline{AD}$

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta BAC$	
$\hat{D} \cong \hat{C}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{AB} \cong \overline{BA}$	Common
$\therefore \quad \Delta ABD \cong \Delta BAC$	H.S. ≅ H.S.
So $\quad \overline{AC} \cong \overline{BD}$	Corresponding sides of ≅ Δ's.
and $\quad \angle BAC \cong \angle ABD$	Corresponding sides of ≅ Δ's.

Q1. **Which of the following are true and which are false? ; Review EX #10 ; Q.1**

(i) A ray has two end points.
(ii) In a triangle, there can be only right angle.
(iii) Three points are said to be collinear if they lie on same line.
(iv) Two parallel lines intersect only at a point.
(v) Two lines can intersect only at one point.
(vi) A triangle of congruent sides has non-congruent angles.

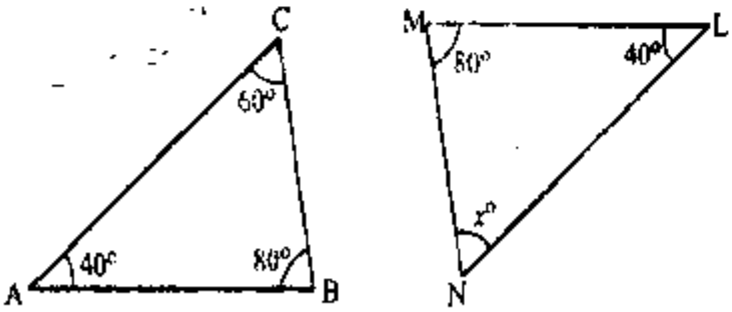
Answers:

(i) F	(ii) T	(ii) T	(iv) F	(v) T	(vi) F

Review EX #10 ; Q.3

Q3. **If $\Delta ABC = \Delta LMN$, then find the unknown x.**

Solution:



Given that $\quad \Delta ABC = \Delta LMN$

$\therefore \quad \angle C \cong \angle M$

or $\quad m\angle C \cong m\angle M$

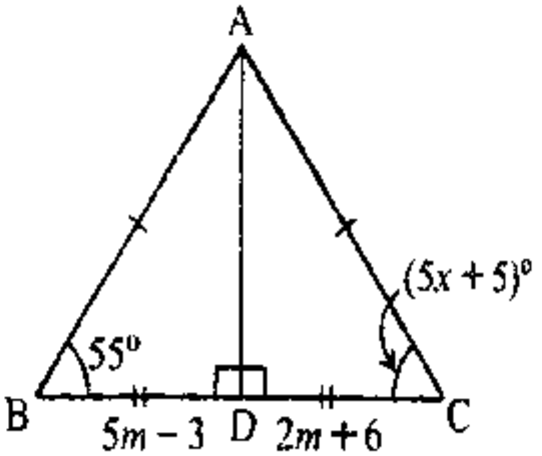
$\Rightarrow \quad 60^\circ = x^\circ$

$\Rightarrow \quad x = 60^0$

Review EX #10 ; Q.4

Q4. **Find the value unknowns for the given congruent triangles.**

Solution:



$\Delta ADB \cong \Delta ADC$

$\overline{BD} \cong \overline{CD}$

Corresponding sides of ≅ Δ's.

$\Rightarrow \quad m\overline{BD} \cong m\overline{CD}$

$\Rightarrow \quad 5m - 3 = 2m + 6$

or $\quad 5m - 2m = 6 + 3$

$3m = 9$

$\angle B \cong \angle C$

Corresponding sides of ≅ Δ's.

$\Rightarrow \quad m\angle B \cong m\angle C$

$55^\circ = (5x + 5)^\circ$

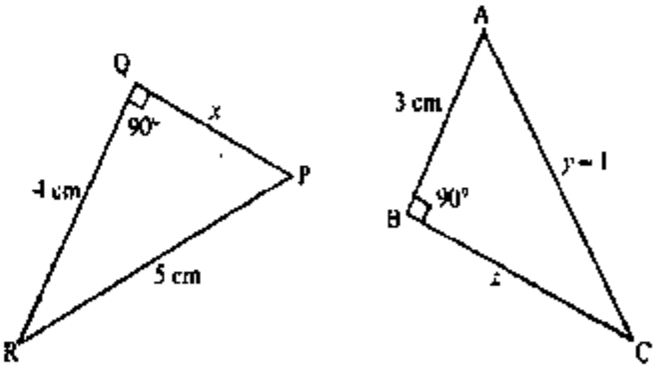
$\Rightarrow \quad 55 = 5x + 5$

$\Rightarrow \quad 5x = 55 - 5 = 50$

Review EX #10 ; Q.5

Q5. If PQR ≅ ABC, then find the unknowns.

Solution:



$\Delta PQR \cong \Delta ABC$

$\therefore \quad \overline{BD} \cong \overline{CD}$

Corresponding sides of ≅ Δ's.

$\Rightarrow \quad x = 3\text{ cm}$

$\overline{PR} \cong \overline{AC}$

Corresponding sides of ≅ Δ's.

$\Rightarrow \quad 5 = y - 1 \quad \Rightarrow \quad y = 5 + 1$

$\Rightarrow \quad y = 6$

Also $\overline{QR} \cong \overline{BC}$

Corresponding sides of ≅ Δ's.

$m\overline{QR} \cong m\overline{BC} \quad \Rightarrow \quad 4\text{ cm} = z \Rightarrow \text{ or } \quad z = 4\text{ cm}$

Review EX #11 ; Q.1

Q1. Fill in the blanks.

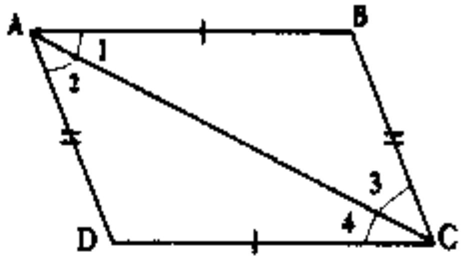
(i) In a parallelogram opposite sides are ....
(ii) In a parallelogram opposite angles are ....
(iii) Diagonals of a parallelogram .... each other at a point.
(iv) Medians of a triangle are ....
(v) Diagonals of a parallelogram divides the parallelogram into two .... triangles.

Answers:

(i) parallel/congruent (ii) equal/congruent (iii) intersect
(iv) concurrent (v) congruent

Review EX #11 ; Q.2

Q2. In parallelogram ABCD



(i) $m\overline{AB}$ ..... $m\overline{DC}$ (ii) $m\overline{BC}$ ..... $m\overline{AD}$
(iii) $m\angle 1 \cong$ ..... (iv) $m\angle 2 \cong$ .....

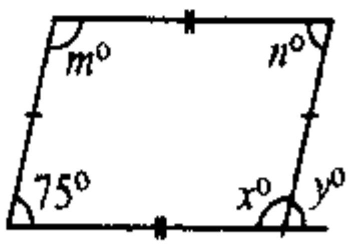
Answers:

(i) ≅ (ii) ≅
(iii) $m\angle 3$ (iv) $m\angle 1$

Review EX #11 ; Q.3

Q3. Find the unknowns in the given figure.

Solution:



$n° \cong 75°$ opposite angles are congruent

$\therefore \quad n = 75$

$y° \cong n°$ Alternate angles

$y° \cong n° \cong 75°$

$\therefore \quad y = 45$

$x° + y° = 180°$ Supplementary angles

$x + y = 180$

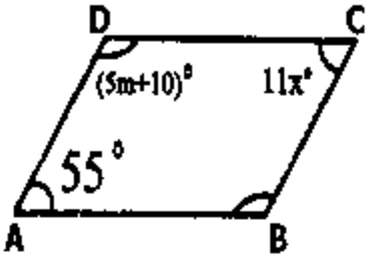
$x + 75 = 180$

$x = 180 - 75 = 105$

Review EX #11 ; Q.4

Q4. If the given figure ABCD is a parallelogram, then find x, m.

Solution:



$11x^\circ \cong 55^\circ$ opposite angles

$11x = 55$

$x = 5^0$

$(5m + 10)^\circ + 55^\circ = 180^\circ$

Sum of interior angles of ∥ lines

$5m + 10 + 55 = 180$

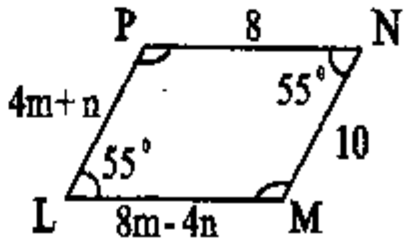
$5m + 65 = 180$

or $m = 23^0$

Review EX #11 ; Q.5

Q5. The given figure LMNP is a parallelogram. Find the value of m, n.

Solution:



As opposite sides of a parallelogram are congruent

$8m - 4n = 8$

or $2m - n = 2$ (i)

and $4m + n = 10$ (ii)

Adding (i) and (ii)

$6m = 12$

or $m = 2$

Putting $m = 2$ in (i) we have

$2(2) - n = 2$

$4 - n = 2$

or $-n = -2$ or $n = 2$

$\therefore$ $m = 2, n = 2$

Review EX #12 ; Q.1

Q1. Which of the following are true and which are false?

(i) Bisection means to divide into two parts.

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point of line segment.

(iii) Any point on the right bisector of a line segment is not equidistant from its end points.

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it.

(v) The right bisector of the sides of a triangle is not concurrent.

(vi) The bisectors of the angles of a triangle are concurrent

(vii) Any point on the bisector of an angle is not equidistant from its arm.

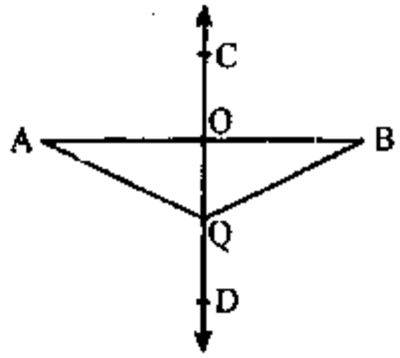
(viii) Any point inside an angle, equidistant from its arms, is on the bisector of it.

Answers:

(i) T	(ii) T	(iii) F	(iv) T
(v) F	(vi) T	(vii) F	(viii) T

Review EX #12 ; Q.2

Q2. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then



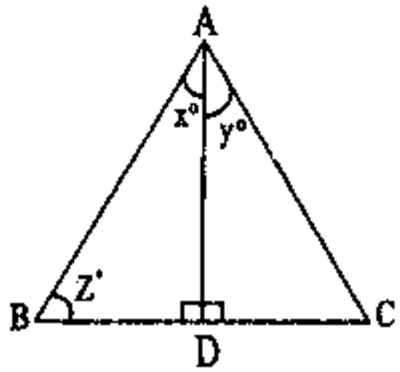
(i) $m\,\overline{OA}=$........ (ii) $m\,\overline{AQ}=$........

Answers:

(i) $m\,\overline{OB}$	(ii) $m\,\overline{BQ}$

Review EX #12 ; Q.4

Q4. The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknown x^0, y^0 and z^0.



Solution:

ΔABC is equilateral

$\therefore\ m\angle A = m\angle B = m\angle C = 60^o$

$\therefore\ z^o = 60^o$

$\overline{AD}$ is bisector of $\angle A$

$\therefore\ x^o = y^o = \frac{1}{2} m\angle A$

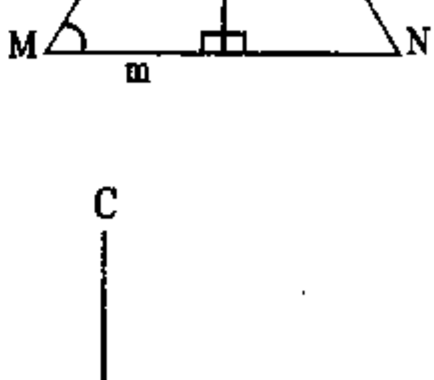
$= \frac{1}{2}(60^o) = 30^o \therefore\ x^0 = y^0 = 30^0$

Review EX #12 ; Q.5

Q5. In the given congruent triangles LMO and LNO, find the unknowns x and m.

Solution:

Corresponding sides of congruent triangles ΔLMO and ΔLNO.



$\overline{LM} \cong \overline{LN}$

$\therefore\ 2x+6=18$

$\Rightarrow\ 2x = 18-6 = 12$

$\therefore\ x = \frac{12}{6} = 6$

Given that $m\,\overline{ON} = 12$

Since given triangles are congruent therefore

$m\,\overline{OM} = m\,\overline{ON} = 12$

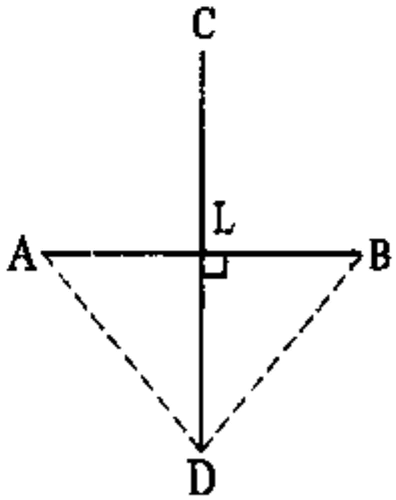
$m\,\overline{OM} = m = 12$

Review EX #12 ; Q.6

Q6. $\overline{CD}$ Is the right bisector of the line segment AB.

(i) If $m\,\overline{AB} = 6\,cm$, then find the $m\,\overline{AL}$ and $m\,\overline{LB}$

(ii) If $m\,\overline{BD} = 4\,cm$, then find the $m\,\overline{AD}$



Solution: CD is right bisector

$\therefore\ \overline{AL} \cong \overline{BL}$

$\therefore\ m\overline{AL} = m\overline{BL}$

$= \frac{1}{2}(m\overline{AB}) \qquad = \frac{1}{2}(6\,cm) = 3cm$

$\therefore\ m\overline{AL} = m\overline{BL} = 3\,cm$

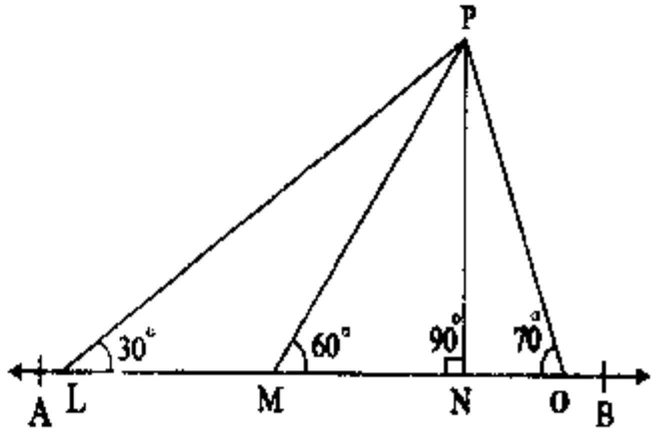
In $\Delta ALD \leftrightarrow \Delta BLD$

$\overline{AL} \cong \overline{BL}$

EX #13.2 ; Q.1

Q1. In the figure, P is any point and AB is a line. Which of the following is the shortest distance between the point P and the line AB.

(a) $m\overline{PL}$ (b) $m\overline{PM}$
(c) $m\overline{PN}$ (d) $m\overline{PO}$



Solution:

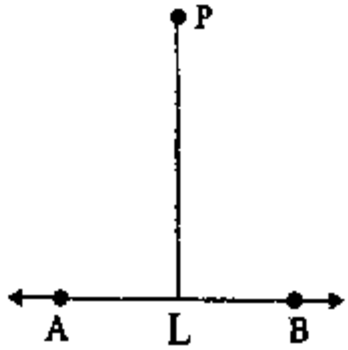
We know that form a point outside a line, the perpendicular is the shortest distance from the point to the line.

As $\overline{PN}$ is perpendicular to AB
So $\overline{PN}$ is the shortest distance.

EX #13.2 ; Q.2

Q2. In the figure, P is any point lying away from the line AB. Then $m\overline{PL}$ will be the shortest distance if

(a) $m\angle PLA = 80^\circ$
(b) $m\angle PLB = 100^\circ$
(c) $m\angle PLA = 90^\circ$



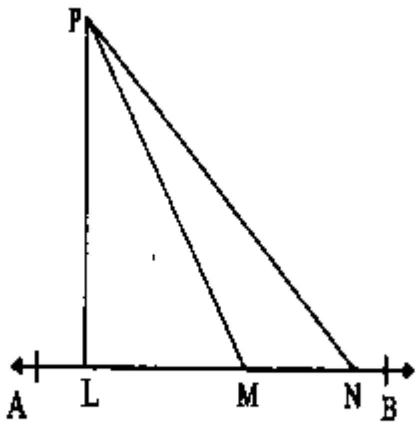
Solution: We know that for a point outside a line, the shortest distance from the point to the line is perpendicular to the line.

As $m\overline{PL}$ is shortest,
So $\overline{PL}$ is perpendicular to $\overline{AB}$.
So $m\angle PLA = 90^\circ$

EX #13.2 ; Q.3

Q3. In the figure, $\overline{PL}$ is perpendicular to the line AB and $m\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.

Solution:



Given:

$\overline{PL}$ is perpendicular to $\overleftrightarrow{AB}$ and $m\overline{LN} > m\overline{LM}$.

To prove:

$m\overline{PN} > m\overline{PM}$

Proof:

Statements	Reasons
In ΔLPN	
$m\angle PLN = 90^\circ$	Given
$\therefore$ $m\angle PLN < 90^\circ$ (i)	Angle of a triangle
In ΔPLM	
$m\angle PMN > m\angle PLM$	Exterior angle
$\therefore$ $m\angle PMN < 90^\circ$ (ii)	$\angle PLM = 90^\circ$
In ΔPMN	

Q1. Which of the following are true and which are false? ; Review EX #13 ; Q.1

(i) The angle opposite to the longer side is greater.
(ii) In a right-angled triangle greater angle is of 60°.
(iii) In an isosceles right-angled triangle, angles other than right angle are each of 60°.
(iv) A triangle having two congruent sides is called equilateral triangle.
(v) A perpendicular from a point to line is shortest distance.
(vi) Perpendicular to line form an angle of 60°.
(vii) A point outside the line is collinear.
(viii) Sum of two sides of triangle is greater than the third.
(ix) The distance between a line a point on it is zero.
(x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm.

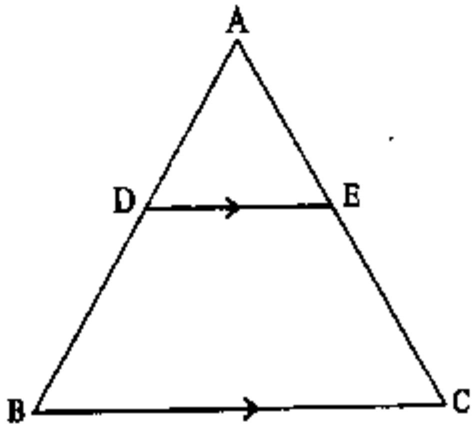
Solution:

(i) T	(ii) F	(iii) T	(iv) F	(v) T
(vi) T	(vii) F	(viii) T	(ix) T	(x) F

EX #14.1; Q.1

Q1. In ΔABC, $\overline{DE} \parallel \overline{BC}$.

Solution:



(i) $\overline{AD} = 1.5$ cm, $\overline{BD} = 3$ cm, $\overline{AE} = 1.3$ cm, $\overline{CE} = ?$

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\therefore \quad \dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$\therefore \quad \dfrac{1.5}{3} = \dfrac{1.3}{m\overline{EC}}$

$\Rightarrow \quad 1.5(m\overline{EC}) = (1.3)3$

$m\overline{EC} = \dfrac{1.3\times1.3}{1.5} = \dfrac{13\times3\times10}{10\times15}$

$= \dfrac{13\times3}{15} = \dfrac{13}{5} = 2.6$ cm

(ii) $\overline{AD} = 2.4$ cm, $\overline{AE} = 3.2$ cm, $\overline{EC} = 4.8$ cm, $\overline{AB} = ?$

In ΔABC, $\overline{DE} \parallel \overline{BC}$; As $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$

$\therefore \quad \dfrac{2.4}{m\overline{DB}} = \dfrac{3.2}{4.8}$

$3.2(m\overline{DB}) = (2.4)(4.8)$

$m\overline{DB} = \dfrac{(2.4)(4.8)}{3.2} = \dfrac{24\times10\times48}{10\times32\times10} = \dfrac{36}{10} = 3.6$ cm

$m\overline{AB} = m\overline{AD} + m\overline{DB} = 2.4 + 3.6 = 6.0$ cm

(iii) $\dfrac{\overline{AD}}{\overline{DB}} = \dfrac{3}{5}$, $\overline{AC} = 4.8$, $\overline{AE} = ?$; In ΔABC, $\overline{DE} \parallel \overline{BC}$

As $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{m\overline{AE}}{m\overline{EC}}$; $\therefore \quad \dfrac{3}{5} = \dfrac{\overline{AE}}{\overline{EC}}$

$\therefore \quad \dfrac{3}{5} + 1 = \dfrac{\overline{AE}}{\overline{EC}} + 1$; $\dfrac{8}{5} = \dfrac{\overline{AE}+\overline{EC}}{\overline{EC}} = \dfrac{\overline{AC}}{\overline{EC}}$

$\dfrac{8}{5} = \dfrac{4.8}{\overline{EC}} \Rightarrow 8\overline{EC} = 4.8 \times 5 = 24$

$\overline{EC} = \dfrac{24}{8} = 3$

$\frac{\overline{AD}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AC}} = \frac{\overline{DE}}{\overline{BC}} \Rightarrow \therefore \frac{2.4}{\overline{AB}} = \frac{3.2}{\overline{AC}} = \frac{2}{5}$

$2(\overline{AB}) = 5(2.4) = 12$

$\overline{AB} = \frac{12}{2} = 6\text{ cm} \Rightarrow 2(\overline{AC}) = (3.2)5 = 16$

$\overline{AC} = \frac{16}{2} = 8\text{ cm} \Rightarrow \overline{DE} = \overline{AB} - \overline{AD} = 6 - 2.4 = 3.6\text{ cm}$

$\overline{CE} = \overline{AC} - \overline{AE} = 8 - 3.2 = 4.8\text{ cm}$

(v) $\overline{AD} = 4x - 3$, $\overline{AE} = 8x - 7$, $\overline{BD} = 3x - 1$, $\overline{CE} = 5x - 3$, find x.

In ΔABC, $\overline{DE} \parallel \overline{BC}$

$\frac{\overline{AD}}{\overline{BD}} = \frac{\overline{AE}}{\overline{EC}} \Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$(4x-3)(5x-3) = (8x-7)(3x-1)$

$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$ or $20x^2 - 24x^2 - 27x + 29x + 9 - 7 = 0$

$-4x^2 + 2x + 2 = 0$ or $2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0 \Rightarrow 2x(x-1) + (x-1) = 0$. $(x-1)(2x+1) = 0$

$x = 1, -\frac{1}{2}$; For $x = -\frac{1}{2}$ sides become negative. ; So $x = 1$

EX #14.2; Q.1

Q1. In ΔABC as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. $m\overline{BD}$ is equal to

(a) 5 (b) 16 (c) 10 (d) 18

Solution:

In ΔABC, $\overline{CD}$ bisect $\angle C$ meets $\overline{AB}$ at D.

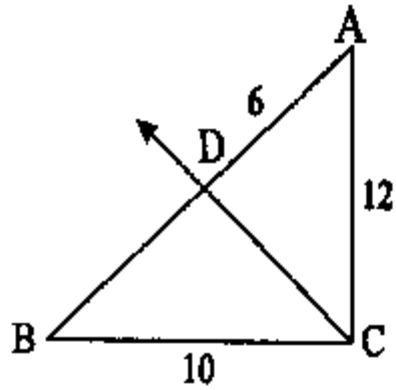
As $\overline{CD}$ is the internal bisector of $\angle C$

So $\frac{m\overline{BD}}{m\overline{DA}} = \frac{m\overline{BC}}{m\overline{CA}}$

$\frac{m\overline{BD}}{6} = \frac{10}{12}$

$m\overline{BD} = 6 \times \frac{10}{12} = 5$

So the correct answer is (a).



EX #14.2; Q.2

Q2. In ΔABC shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.

Solution:

In ΔABC, $m\overline{AC} = 3$

$m\overline{CB} = 6$, $m\overline{AB} = 7$

Let $m\overline{AD} = x$

then $m\overline{DB} = 7 - x$

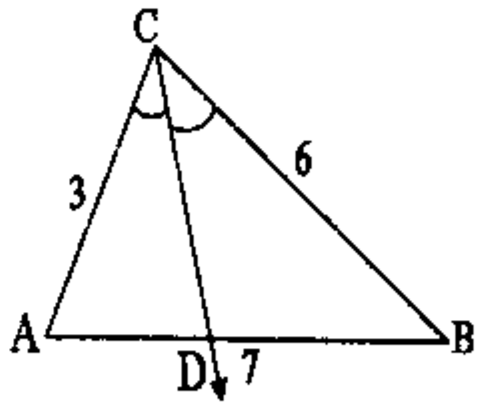
As $\overrightarrow{CD}$ is internal bisector of $\angle C$

So $\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{CB}} \Rightarrow \frac{x}{7-x} = \frac{3}{6}$

$6x = 21 - 3x \Rightarrow 9x = 21$

$x = \frac{21}{9} \Rightarrow m\overline{AD} = \frac{21}{9} = \frac{7}{3}$

$m\overline{DB} = m\overline{AB} - m\overline{AD} \Rightarrow m\overline{DB} = 7 - \frac{21}{9} = \frac{63-21}{9} = \frac{42}{9} = \frac{14}{3}$



Q1. Which of the following are true and which are false? Review EX #14 ; Q.1

(i) Congruent triangles are of same size and shape.

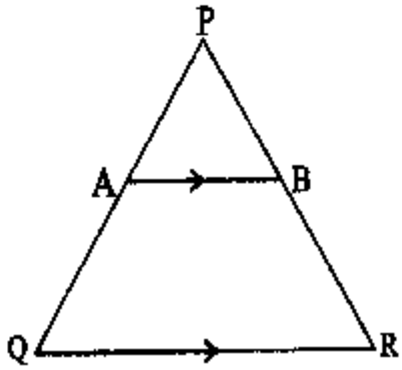
(v) Congruent triangles are similar.
(vi) Similar triangles are congruent.
(vii) A line segment has only one mid-point.
(viii) One and only one line can be drawn through two points.
(ix) Proportion is non-equality of two ratios.
(x) Ratio has no unit.

Answers:

(i) T	(ii) T	(iii) F	(iv) F	(v) T
(vi) F	(vii) T	(viii) T	(ix) F	(x) T

Review EX #14 ; Q.4

Q4. In the shown figure, let $m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$, $m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$. Find the value of x if $\overline{AB} \parallel \overline{QR}$.



Solution:

$m\overline{PA} = 8x - 7$, $m\overline{PB} = 4x - 3$

$m\overline{AQ} = 5x - 3$, $m\overline{BR} = 3x - 1$

As $\overline{AB} \parallel \overline{QR}$

$\frac{m\overline{PA}}{m\overline{AQ}} = \frac{m\overline{PB}}{m\overline{BR}}$

$\therefore \frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$(8x - 7)(3x - 1) = (4x - 3)(5x - 3)$

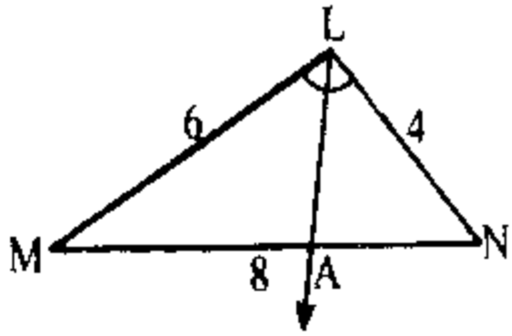
$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9 \Rightarrow \quad 24x^2 - 20x^2 - 29x + 27x + 7 = 9$

$4x^2 - 2x + 7 = 9 \Rightarrow 4x^2 - 2x - 2 = 0 \Rightarrow 2x^2 - x - 1 = 0 \Rightarrow \quad 2x^2 - 2x + x - 1$

$2x(x - 1) + (x - 1) = 0 \Rightarrow \quad (x - 1)(2x + 1) = 0 \quad x = 1, -\frac{1}{2}$; $x = 1$ is the required value.

Q5. In Δ LMN show in the figure, $\overrightarrow{LA}$ bisect $\angle L$. If $m\overline{LN} = 4$ $m\overline{LM} = 6$, $m\overline{MN} = 8$, then find $m\overline{MA}$ and $m\overline{AN}$. **Review EX #14 ; Q.5**

Solution:



$m\overline{LN} = 4$, $m\overline{LN} = 6$, $m\overline{MN} = 8$

$\overrightarrow{LA}$ is bisector of $\angle L$

$\therefore \frac{m\overline{MA}}{m\overline{NA}} = \frac{m\overline{LM}}{m\overline{LN}} = \frac{6}{4}$

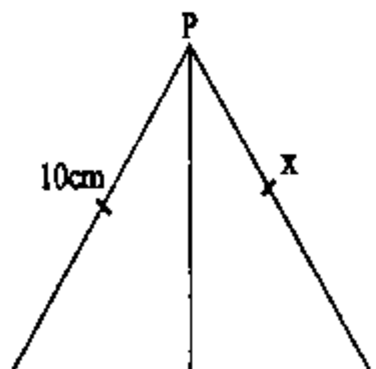
i.e. $m\overline{MA} : m\overline{NA} = 6 : 4$

but $m\overline{MN} = m\overline{MA} + m\overline{AN} = 8$

$\therefore m\overline{MA} = \frac{6}{10} \times 8 = \frac{48}{10} = 4.8$

and $m\overline{AN} = \frac{4}{10} \times 8 = \frac{32}{10} = 3.2$

Q6. In isosceles ΔPQR shown in the figure, find the value of x and y. **Review EX #14 ; Q.6**



Solution:

$\overline{PQ} \cong \overline{PR}$

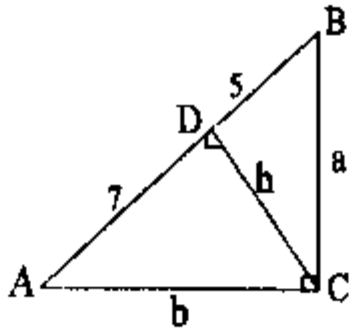
$\Rightarrow$ x = 10 cm

PM $\perp$ QR where PQR is an isosceles triangle

$\therefore m\overline{MQ} = m\overline{MR}$

EX #15; Q.6

Q6. (i) In the ΔABC as shown in the figure, $m\angle ACB = 90^0$ and $\overline{CD} \perp \overline{AB}$. Find the lengths a, h and b if $m\overline{BD} = 5\ units$ and $m\overline{AD} = 7\ units$



Solution: $m\overline{AB} = 5 + 7 = 12$

In right angled ΔBDC

$a^2 = 25 + h^2$ .... (1)

In right angled ΔADC

$b^2 = 49 + h^2$ .... (2)

In right angled ΔABC

$a^2 + b^2 = 144$ ..... (3)

Adding (1) and (2)

$a^2 + b^2 = 74 + 2h^2$...... (4)

From (3) and (4) $\quad 74 + 2h^2 = 144$

$2h^2 = 144 - 74 = 70 \Rightarrow \quad h^2 = 35 \quad \Rightarrow h = \sqrt{35}\ units$

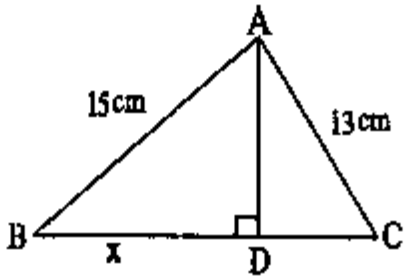
Put $h^2 = 35$ in (1) ; $\quad a^2 = 25 + 35 = 60$

$a = \sqrt{60} = 2\sqrt{15}\ units$; Put $h^2 = 35$ in (2) ; $\quad b^2 = 49 + 35 \Rightarrow \quad b^2 = 84$

$b = \sqrt{84} = 2\sqrt{21}\ units$; So $a = 2\sqrt{15}\ units$; $\quad h = \sqrt{35}\ units$; $\quad b = 2\sqrt{21}\ units$

(ii) Find the value of x in the shown figure.

Solution:



From ΔADC

$(m\overline{AC})^2 = (m\overline{AD})^2 + (m\overline{DC})^2$

$(13)^2 = (m\overline{AD})^2 + (5)^2$

$169 = (m\overline{AD})^2 + 25$

$(m\overline{AD})^2 = 169 - 25 = 144$

$\therefore \quad m\overline{AD} = 12cm$

From ΔABD ; $\quad (m\overline{AB})^2 = (m\overline{AD})^2 + (m\overline{BD})^2$

$(15)^2 = (12)^2 + (x)^2 \Rightarrow \quad 225 = 144 + x^2 \Rightarrow \quad x^2 = 225 - 144 = 81 \Rightarrow$

$\therefore \quad x = 9\ cm$

Q1. Which of the following is true and which are false? ; Review EX #15 ; Q.1

(i) In a right angled triangle greater angle is of 90^0.

(ii) In a right angled triangle right angle is of 60^0.

(iii) In a right triangle hypotenuse is a side opposite to right angle.

(iv) If a, b, c are sides of right angled triangle with c as longer side then $c^2 = a^2 + b^2$

(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm.

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm then each of other side is of length 2 cm.

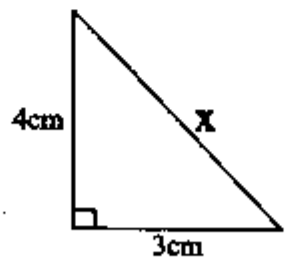
Answers:

(i) T	(ii) F	(iii) T	(iv) T	(v) T	(vi) F

Review EX #15 ; Q.2

Q2. Find the unknown value in each of the following figures.

(i)



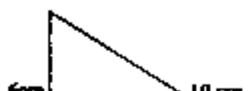
By Pythagoras Theorem

$x^2 = 4^2 + 3^2 = 16 + 9 = 25$

$x^2 = \sqrt{25} = 5\ cm$

(ii)

By Pythagoras Theorem

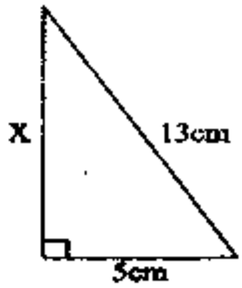


$(10)^2 = (6)^2 + (x)^2$

(iii)

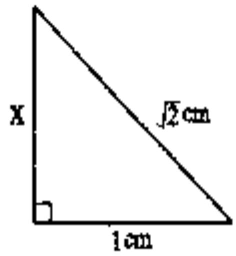
By Pythagoras Theorem

$$(13)^2 = (x)^2 + (5)^2$$
$$169 = x^2 + 25$$
$$x^2 = 169 - 25$$
$$x = \sqrt{144} = 12\ cm$$



(iv) By Pythagoras Theorem

$$\left(\sqrt{2}\right)^2 = (x)^2 + (1)^2$$
$$2 = x^2 + 1$$
$$x^2 = \sqrt{1} = 1\ cm$$



Review EX #16 ; Q.1

Q1. Which of the following are true and which are false?

- **(i)** Area of a figure means region enclosed by bounding lines of closed figure.
- **(ii)** Similar figure have same area.
- **(iii)** Congruent figures have same area.
- **(iv)** A diagonal of a parallelogram divides it into two non-congruent triangles.
- **(v)** Altitude of a triangle means perpendicular from vertex to the opposite side (base).
- **(vi)** Area of parallelogram is equal to the product of base and height.

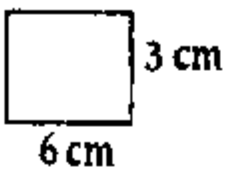
Answers:

(i) T	(ii) F	(iii) T	(iv) F	(v) T	(vi) T

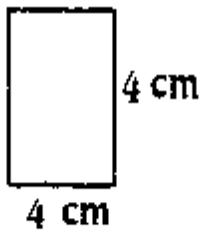
Review EX #16 ; Q.2

Q2. Find the area of the following.

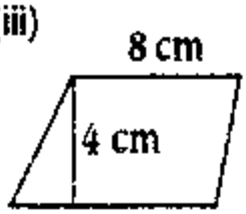
(i)



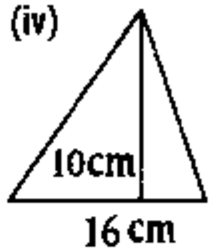
(ii)



(iii)



(iv)



Solution:

(i) Area of rectangle $=$ Length $\times$ Width $= 6 \times 3 = 18\ \text{cm}^2$

(ii) Area of square $=$ Side $\times$ Side $= 4 \times 4 = 16\ \text{cm}^2$

(iii) Area of rectangle $=$ Length $\times$ Width $= 8 \times 4 = 32\ \text{cm}^2$

(iv) Area of triangle $= \frac{1}{2}$ base $\times$ altitude $= \frac{1}{2} \times 10 \times 16 = 80\ \text{cm}^2$

EX #17.1 Q.1;(i)

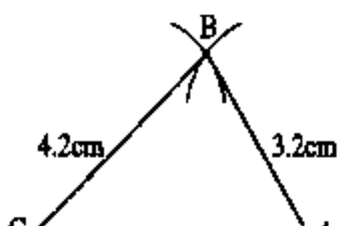
Q1. Construct a ΔABC, in which

(i) $m\overline{AB} = 3.2$ cm, $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 5.2$ cm

Solution:

Construction:

(i) Draw a line segment $m\overline{CA} = 5.2$ cm



(iv) Join BC and AB.
Then ABC is the required triangle.

EX #17.1 Q.1;(iv)

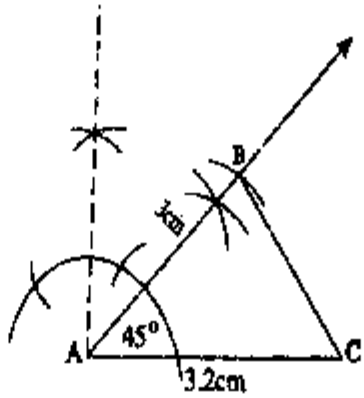
Q1. Construct a ΔABC, in which

(iv) $m\overline{AB} = 3$ cm, $m\overline{AC} = 3.2$ cm, $m\angle A = 45^\circ$

Solution:

Construction:

(i) Draw a line segment $m\overline{AC} = 3.2$ cm.
(ii) At the end A of $\overline{AC}$ make $\angle CAB = 45^\circ$.
(iii) Cut off $m\overline{AB} = 3$ cm.
(iv) Join B to C
So ABC is the required triangle



EX #17.1 Q.1;(v)

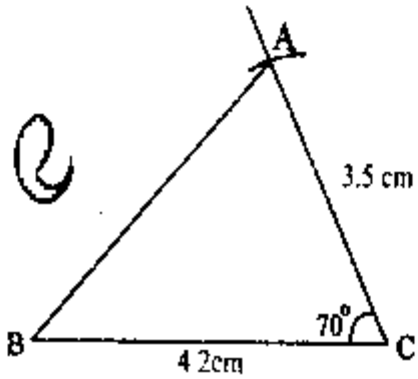
Q1. Construct a ΔABC, in which

(v) $m\overline{BC} = 4.2$ cm, $m\overline{CA} = 3.5$ cm, $m\angle C = 75^\circ$

Solution:

Construction:

(i) Draw a line segment $m\overline{BC} = 4.2$ cm.
(ii) At the end C of $\overline{BC}$ make $\angle BCA = 70^\circ$.
(iii) Cut off $m\overline{CA} = 3.5$ cm.



EX #17.1 Q.2;(i)

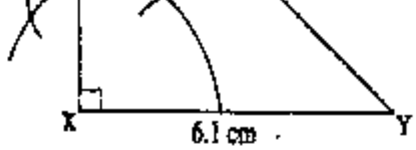
Q2. Construct a ΔXYZ, in which

(i) $m\overline{YZ} = 7.6$ cm, $m\overline{XY} = 6.1$ and $m\angle X = 90^\circ$

Solution:

Construction:

(i) Draw a line segment $m\overline{XY} = 6.1$ cm.
(ii) At the end point X of $\overline{XY}$ make $\angle YXL = 90^\circ$.
(iii) With centre Y and radius equal to 7.6 cm draw an arc to cut $\overrightarrow{XL}$ at point Z.
(iv) Join T to Z.
Then XYZ is the required Δ.



EX #17.1 Q.2;(ii)

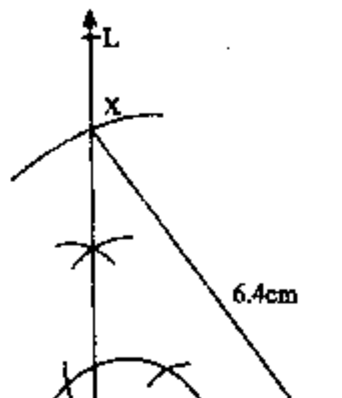
Q2. Construct a ΔXYZ, in which

(ii) $m\overline{ZX} = 6.4$ cm, $m\overline{YZ} = 2.4$ and $m\angle Y = 90^\circ$

Solution:

Construction:

(i) Draw a line segment $m\overline{YZ} = 2.4$ cm.
(ii) At the end point Y of $\overline{YZ}$ make $\angle XYZ = 90^\circ$.
(iii) With centre Z and radius equal to 6.4 cm draw an arc to cut $\overrightarrow{YL}$ at point X.
(iv) Join X to Z.



EX #17.1 Q.4;(ii)

Q4. Construct a right-angled isosceles triangle whose hypotenuse is

(ii) 4.8 cm

Solution:

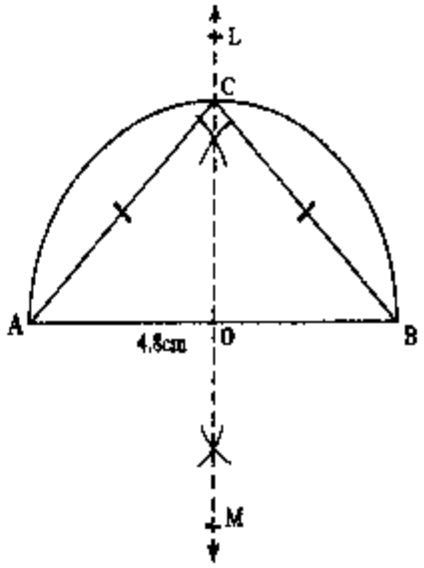
Construction:

(i) Draw a line segment $m\overline{AB} = 4.8$ cm.

(ii) Draw $\overleftrightarrow{LM}$ the right bisector of $\overline{AB}$ cutting it at the point O.

(iii) With O as centre and $\overline{AB}$ as diameter draw a semi-circle to cut $\overleftrightarrow{LM}$ at the point C.

(iv) Join C to A and B.
Then the required triangle is ABC.



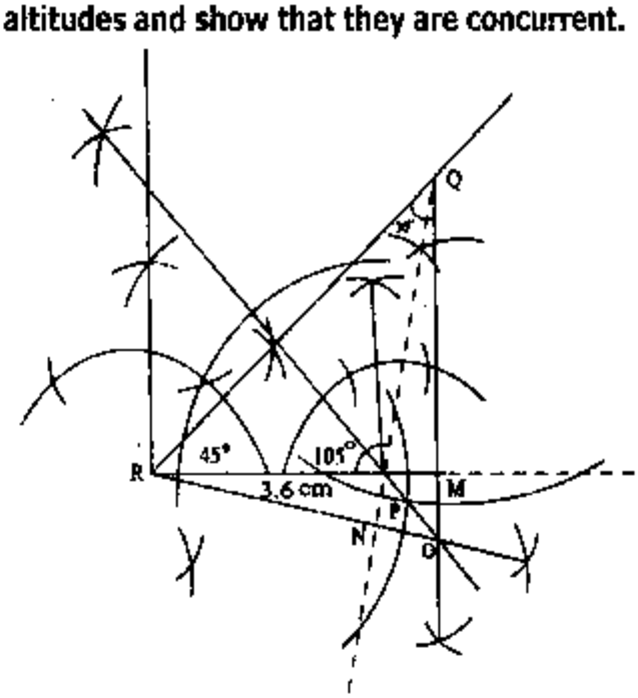
EX #17.2 Q.2;(iii)

Q2. Construct the following Δ's PQR. Draw their altitudes and show that they are concurrent.

(iii) $m\overline{RP} = 3.6$ cm, $m\angle Q = 30°$, $m\angle P = 105°$

Solution:

$$m\angle Q = 30°, m\angle P = 105°$$
$$m\angle P + m\angle Q + m\angle R = 180°$$
$$105° + 30° + m\angle R = 180°$$
$$m\angle R = 180° - 135 = 45°$$



Construction:

(i) Take $m\overline{RP} = 3.6$ cm.

(ii) Draw $m\angle QRS = 45°$ and $m\angle RPQ = 105°$ to complete ΔPQR.

(iii) From the vertex P drop $\overline{PL} \perp \overline{QR}$.

(iv) From the vertex Q drop $\overline{QM} \perp \overline{RP}$ produced.
These two altitudes meet at the point O.

(v) Now from the third vertex R drop $\overline{RN} \perp \overline{QP}$ produced.

(vi) We observe that the third altitude also passes through the point of intersection O of the first two altitudes.

(vii) Hence the three altitudes of ΔPQR are concurrent at O.

EX #17.2 Q.3;(iii)

Q3. Construct the following triangles ABC. Draw the perpendicular bisectors of their sides and verify their concurrency. Do you meet inside the triangle?

(iii) $m\overline{AB} = 2.4$ cm, $m\overline{CA} = 3.2$ cm, $m\angle A = 120°$

Solution:

Construction:

(i) Take $m\overline{AB} = 2.4$ cm.

(v) Draw perpendicular bisectors of $\overline{BC}$ and $\overline{CA}$ meeting each other at the point O.
(vi) Now draw the perpendicular bisector of third side $\overline{AB}$.
(vii) We observe that it also passes through O, the point of intersection of first two perpendicular bisectors.
(vii) Hence the three perpendicular bisectors of ΔABC are concurrent at O.

Q1. Fill in the following blanks to make the statement true: Review EX #17 Q.1

(i) The side of a right angled triangle opposite to 90° is called.....
(ii) The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a......
(iii) A line drawn from a vertex of a triangle which is ..... to its opposite side is called an altitude of the triangle.
(iv) The bisectors of the three angles of a triangle are ....
(v) The point of concurrency of the right bisectors of the three sides of the triangle is......from its vertices.
(vi) Two or more triangles are said to be similar if they are equiangular and measures of their corresponding sides are.......
(vii) The altitudes of a right triangle are concurrent at the......of the right angle.

Answers:

(i) hypotenuse (ii) median (iii) perpendicular
(iv) concurrent (v) equidistant (vi) proportional
(vii) vertex

Q2. Multiple Choice Questions. Choose the correct answer. ; Review EX #17 Q.2

(i) A triangle having two sides congruent is called.....
(a) scalene (b) right angled (c) equilateral (d) isosceles

(ii) A quadrilateral having each angle equal to 90° is called......
(a) parallelogram (b) rectangle (c) trapezium (d) rhombus

(iii) The right bisectors of the three sides of a triangle are......
(a) congruent (b) collinear (c) concurrent (d) parallel

(iv) The......altitudes of an isosceles triangle are congruent.
(a) two (b) three (c) four (d) none

(v) A point equidistant from the end points of a line-segment is on its......
(a) bisector (b) right-bisector (c) perpendicular (d) median

(vi) ....congruent triangles can be made by joining the mid-points of the sides of a triangle.
(a) three (b) four (c) five (d) two

(vii) The diagonals of a parallelogram......each other.
(a) bisect (b) trisect (c) bisect at right angle (d) none of these

(viii) The medians of a triangle cut each other in the ratio......
(a) 4 : 1 (b) 3 . 1 (c) 2 1 (d) 1:1

(ix) One angle on the base of an isosceles triangle is 30°. What is the measure of its vertical angle.......
(a) 30° (b) 60° (c) 90° (d) 120°

(x) If the three altitudes of a triangle are congruent, then the triangle is......
(a) equilateral (b) right angled (c) isosceles (d) acute angled

(xi) It two medians of a triangle are congruent then the triangle will be......
(a) isosceles (b) equilateral (c) right angled (d) acute angled

Answers:

(i) d	(ii) b	(iii) c	(iv) a	(v) b
(vi) b	(vii) a	(viii) c	(ix) d	(x) a
(xi) a				

Laws of Logarithm

Q1. Prove that $log_a(mn) = log_a m + log_a n$ *(Law of logarithm)*

Solution: Let $log_a m = x,$ $\Rightarrow a^x = m$; *(exponential form)*

and $log_a n = y$ $\Rightarrow a^y = n$; *(exponential form)*

$\therefore a^x \times a^y = mn \quad \Rightarrow a^{x+y} = mn \quad \Rightarrow log_a(mn) = x + y = log_a m + log_a n$

Hence $log_a(mn) = log_a m + log_a n$ **Proved**

Q2. Prove that $log_a\left(\frac{m}{n}\right) = log_a m - log_a n$ *(Law of logarithm)*

Solution: Let $log_a m = x,$ $\Rightarrow a^x = m$; *(exponential form)*

and $log_a n = y$ $\Rightarrow a^y = n$; *(exponential form)*

$\because \frac{a^x}{a^y} = \frac{m}{n} \quad \Rightarrow \quad a^{x-y} = \frac{m}{n} \quad \Rightarrow log_a\left(\frac{m}{n}\right) = x - y = log_a m - log_a n$

Hence $log_a\left(\frac{m}{n}\right) = log_a m - log_a n$ **Proved**

Q3. Prove that $\log(m^n) = n log_a m$ *(Law of logarithm)*

Solution: Let $log_a m^n = x,$ i.e., $a^x = m^n$

and $log_a m = y,$ i.e., $a^y = m$

Then $a^x = m^n = (a^y)^n$

i.e., $a^x = (a^y)^n = a^{yn} \quad \Rightarrow \quad x = ny$

i.e., $log_a m^n = n\ log_a m$

Q4. Prove that: $log_a^n = log_b^n \times log_a^b$ *(Law of logarithm)*

Solution: $log_a^n = log_b^n \times log_a^b$

Let $log_b^n = x$ ....(i)

$n = b^x$ (Exponential form)

$log_a^n = x\, log_a^{b^x}$

$log_a^n = (x)\, log_n^b$